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THEORY OF SOLID AND BRACED ELASTIC ARCHES,*

APPLIED TO ARCH BRIDGES AND ROOFS IN IRON, WOOD, CONCRETE OR OTHER MATERIAL. GRAPHICAL ANALYSIS.

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I.

GENERAL OBSERVATIONS AND THEOREMS.

1. The term *solid arch* is applied, in what follows, to arches having a continuous web connecting the flanges; the term *braced arch*, to such as are braced between the flanges by the usual struts and ties, forming any pattern of open web. As contradistinguished from the voussoir arch, the solid or braced arch is capable of supplying tensile resistances, at any ideal section, when needed; though as all arches are composed of elastic materials the term *elastic arch* is applicable to any one of them. When the solid arch is hinged at one or more points (not exceeding three) there are of course no tensile forces exerted at the hinged joints, so that the hinged joints, if any, must be excepted in the above definition.

If the arch has more than three hinged joints, it is no longer stable, except as a suspension bridge. In the voussoir arch, composed of many stones laid flat against each other, the joints are not hinged; in fact, if the arch ring is of such proportions, that only compressive forces are exerted at each actual joint, the treatment may fall under that pertaining to solid arches, as was first suggested in the preceding paper on *Voussoir Arches*, article 27, remark. This connection between solid and voussoir arches, will be more fully developed in the present paper. The aim has been, in what follows, to treat solid and braced arches in the most general manner; the graphical method offering great facilities in making the discussion very comprehensive, and at the same time simple and obvious. As is well known, the strictly analytical treatment of this subject, especially in the applications, is tedious in the extreme. For the best analytical discussion, the reader is referred to that of Winkler, given in Du Bois' very comprehensive *Graphical Statics*. It is hoped that the purely graphical solution following may be comprehended by many, who posess only a knowledge of the composition and resolution of forces and the principles of moments as taught in mechanics. To this end a brief review of the eqilibrium polygon and some of its properties seems necessary, after which the theory in question will be entered upon.

* It was designed by the Author that this series should immediately follow the articles on *Solid Arches*. The press of other matter prevented.

The Graphical analysis has received attention from Wm. Bell (*Rigid Arches, &c.*, VAN NOSTRAND'S MAGAZINE, Vol. VIII); Prof. Chas. E. Greene (in *Engineering News*, 1877), and Prof. H. T. Eddy in *Researches in Graphical Statics*; much of the latter treatise having first appeared in VAN NOSTRAND'S MAGAZINE for 1877.

The method adopted by Prof. Eddy constitutes a marked advance in precision of treatment, rendering possible the ready and exact location of the true curve of pressures by a systematic method, in place of the method of innumerable trials, heretofore resorted to. It is largely drawn upon in what follows. In order to render the solution as exact as the graphical method will admit of, the few principles of the theory of elasticity that are needed will also be demonstrated.

2. *Equilibrium polygon.*—Let the parallel forces (Fig. 1) acting through r_1, p_1, p_2, p_3 and r_2 be in equilibrium, their intensities being represented, to the scale of force, by the lines R_1, P_1, P_2, P_3 and R_2 on the left, drawn parallel to the forces.

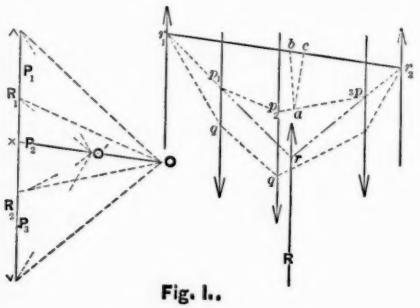

Fig. 1.

From any point O, not on the force line P, draw the dotted rays to the extremities of the lines representing P_1, P_2 . . . . Now commencing at any point r_1 of the first force on the left, draw $r_1 p_1 \parallel$ ray R_1P_1, then $p_1 p_2 \parallel$ ray P_1P_2, $p_2 p_3 \parallel$ ray P_2P_3 and $p_3 r_2 \parallel$ ray P_3R_2; meaning by ray R_1P_1, P_1P_2, . . . , the rays from O included between the letters mentioned.

Now it will be found that the line $r_1 r_2$ is parallel to the ray R_1R_2 or OO′ as may easily be proved from mechanical principles. Thus: conceive applied along each of the lines $r_1 p$, $p_1 p_2$, $p_2 p_3$ and $p_3 r_2$, two opposed forces, each equal to the force that is measured by the length of the ray to which the line is parallel. This does not disturb equilibrium, but it adds at p_1, two forces equal to the rays P_1R_1 and P_1P_2 measured to the scale of force. Similarly at p_2 and p_3 are added the forces represented by the rays P_1P_2, P_2P_3 and P_2P_3, P_3R_2, so that complete equilibrium is established at p_1, p_2 and p_3, since there are three forces at each point proportional to the sides of a triangle parallel to their directions, as we see from the force diagram on the left. Moreover, since the P's act downwards, the applied forces at each vertex, p_1, p_2 . . . , must act *away* from that vertex, as we see by following round the triangle of forces for each vertex p.

Now as the primary forces are in equilibrium, we must likewise have complete equilibrium at both r_1 and r_2 from the original forces R_1 and R_2 and the remaining forces applied along the lines $r_1 p_1$, $p_3 r_2$.

Now in order to have separate equilibrium at both r_1 and r_2 we must apply two opposed forces at r_1 and r_2, each equal to the resultant of R_1 and ray R_1P_1, or of R_2 and ray R_2P_3, (*i. e.* equal to the ray R_1R_2) *which forces balance only when this ray* R_1R_2 *is parallel to* r_1r_2.

Hence, if the forces P_1, P_2, P_3, acting at p_1, p_2, p_3, are given, and it is required to find the forces at r_1 and r_2 (similar to the reactions of a loaded girder) we lay off the forces P_1, P_2 . . . , in order, parallel to their directions; then starting at r_1 or on a vertical through r_1, we draw $r_1p_1 \parallel$ ray R_1P_1, $p_1p_2 \parallel$ ray P_1P_2 etc., to r_2; then a line OO′ drawn parallel to r_1r_2 will divide the force line P into the two reactions R_1 and R_2, for now at each

point, r_1, p_1, . . ., on supplying the imaginary forces we have equilibrium. The line r_1r_2 is called the *closing line*, and the polygon $r_1 p_1 \ldots . r_2$ the *equilibrium polygon*.

Further; if we prolong the sides $r_1 p_1$ and $r_2 p_2$ to intersection r, the resultant $R=R_1+R_2$ acts through r, and is of course parallel to the original forces. This is evident if we consider only the forces at r_1, r_2 acting upwards, and R acting downwards at r, we see that on supplying the opposed forces along the lines rr_1, r_1r_2, r_2r_3 equal to rays P_1R_1, R_1R_2, P_2R that the system is in equilibrium.

3. We shall repeatedly have to solve this problem: given the forces at $p_1, p_2 \ldots$, to find their resultant R. We have then simply, from some point p_1 on the first force, to draw p_1p_2, ∥ ray P_1P_2, &c.; from the last point p_2 so found, draw p_2r ∥ ray P_2R to intersection r with p_1r ∥ ray P_1R. It is seen that on supplying the imaginary forces that the system is in equilibrium at each vertex p_1, $p_2 \ldots p_2$, r. The force R at r, acting in an opposite direction to that which causes equilibrium, is the resultant of the forces at p_1, p_2 . . . in position, direction and magnitude.

4. Again, let us consider the original forces at r_1, p_1, p_2, p_3 and r_2 in conjunction with the forces applied along the sides of the equilibrium polygon $r_1 p_1 p_2 \ldots r_2$ that cause equilibrium at each vertex. Let us suppose a vertical section made at any point a and that the right part abr_2 is removed. Now equilibrium still obtains at the apices r_1, p_1 and p_2. Now for any system of forces in equilibrium, the sum of their movements about any point in their plane is zero. The forces along $r_1 p_1$ and $p_1 p_2$ balance leaving the forces R_1, P_1,P_2, whose moment about a we shall call M, the force along $p_2 p_3$ and that along r_1b which is represented by C. Taking moments about a

$$M-C.\overline{ac}=O$$

Call the perpendicular from the pole O to the force line $P_1 \ldots P_2$ *the pole-distance*=H.

Now $\overline{ac}$ the perpendicular from a upon $r_1r_2=\overline{ab}$. cos. bac. ∴ $C.\overline{ac}=C.$ cos. $bac \times ab$ $=H.\ \overline{ab}$, as is seen from the left figure or force diagram

$$\therefore M=H.ab=Hy.$$

Or the moment of the vertical forces to one side of any point a is equal to the pole distance, H measured to the scale of force multiplied by the ordinate of the equilibrium polygon at that point, to the scale of distance. M is the same taken about any point in the line ab; hence is measured by $H.\overline{ab}$ for any point in the same vertical.

It is this property of the equilibrium polygon that we shall find of special utility in treating solid arches.

As the above result must be true whenever the pole O is placed, we see that for any number of equilibrium polygons the product Hy is constant; *whence a decrease of H involves an increase in y in the same ratio, and the reverse.*

5. If we desire to construct a new equilibrium polygon, with the same closing line r_1r_2 as the first, the new pole O′ must be taken on the ray R_1R_2, since this ray must be parallel to the closing line. The equilibrium polygon is constructed as before, as shown in the figure. If some other pole had been chosen, lying in a vertical line through O′, on drawing the equilibrium polygon through r_1 as before, the point r_2 will be found above or below its present position, but the ordinates y remain the same. If these ordinates are taken in dividers and laid off from the present position of r_1r_2, the equilibrium polygon $qq \ldots$ can be located without the necessity of drawing it directly.

6. If we denote the horizontal distances of the forces R_1,P_1,P_2, acting at r_1,p_1,p_2, from a by x_0,x_1,x_2, respectively, we have just found that,

$$R_1x_0-(P_1x_1+P_2x_2)=H.\overline{ab}=H.y.$$

Produce $\overline{ba}$ to intersection r (it happens to be) with r_1p_1 produced. Now the triangle r_1rb is similar to the one with vertex O and base R_1; ∴ H: R_1:: x_0: $\overline{br}$, or $R_1x_0=H.\overline{br}$. Substituting this value above, we have,

$$P_1x_1+P_2x_2=H.\overline{ar}.$$

From this equation it is evident that we have a ready means of finding the sum of a series of products, of the type Px, in the equilibrium polygon. We have only to draw verticals through points p_1, p_2, whose horizontal distances, to some scale, from the vertical $\overline{ab}$, are x_1,x_2, respect-

ively; then on laying off to some scale, (the same as before if desired) the quantities P_1, P_2, along the line P_1P_2 and constructing an equilibrium polygon through any point r_1, by drawing $r_1p_1 \parallel$ ray R_1P_1 $p_1p_2 \parallel$ ray P_1P_2, $p_2p_3 \parallel$ ray P_2P_3 to intersection a with the vertical $\overline{ab}$, and the closing line r_1a in this case, we find $P_1x_1 + P_2x_2 = H.\overline{ar}$ as before: H being measured to the scale of the P's, and $\overline{ar}$ to the scale of the x's. The principle evidently applies to any number of supposed forces lying to the left of ab. It is evident that the principle is true however near r_1 is taken to p_1. The construction is simplified by causing r_1 and p to coincide so that p_1a will be the closing line. The proof is easily extended to the case when the forces are laid off on $P_1 \ldots P_3$ in any order, and some of them act in opposite directions.

7. The following is a simple geometrical proof that involves the important problem of quadratures. Suppose it required to find the numerical value of the sum of the products $(P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4)$, P_1, x_1, etc., denoting any numerical values whatsoever.

Draw the vertical aa and the horizontal xx (Fig. 2). Through the points $x_1, x_2, \ldots$ distant $x_1, x_2, \ldots$ to scale, from aa draw vertical lines, x_1p_1, etc.

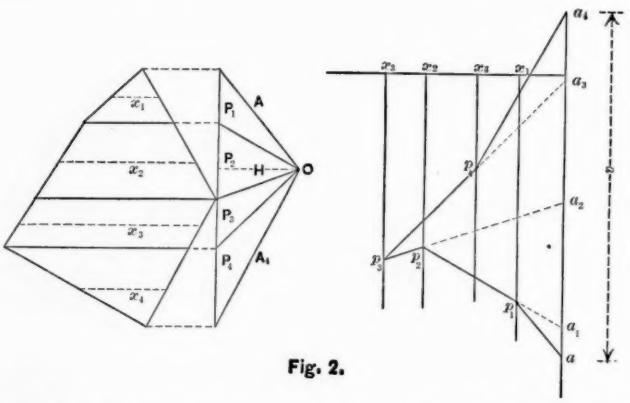

Fig. 2.

Next lay off to some scale (the same scale used for the x's if preferred) $P_1, P_2, \ldots$, in order on a vertical line P_1P_4 and from some point O, draw the rays to the extremities of $P_1, P_2, \ldots$, and denote any ray by the letters enclosing it. Then starting at some point a, draw $ap_1 \parallel$ ray AP_1 to p_1 in the first vertical, then draw $p_1p_2 \parallel$ ray P_1P_2, to intersection p_2 with the vertical p_2, then $p_2p_3 \parallel$ ray P_2P_3, $p_3p_4 \parallel$ ray P_3P_4 and $p_4a_4 \parallel P_4A_4$ to intersection a_4 with the vertical aa. Then denoting by H the perpendicular from O upon the line P_1P_4, measured to the scale of the P's, and measuring $aa_4 = v$, to the scale of the x's, we have

$$P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 = H.v$$

The proof is easy. Extend the lines p_1p_2, p_2p_3, ... to intersection a_1, a_2, ... with aa; then each triangle, p_1aa_1, $p_2a_1a_2$, ... is similar to a triangle in the figure on the left, the sides being parallel; hence the ratio of base to attitude is the same for both

$$\therefore H : P_1 :: x_1 : \overline{aa_1} \quad \therefore P_1x_1 = H.\overline{aa_1}$$
$$\therefore H : P_2 :: x_2 : a_1a_2 \quad \therefore P_2x_2 = H.\overline{a_1a_2}$$
$$\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots$$

Adding the equations, we have

$$P_1x_1 + P_2x_2 + \ldots = H.\overline{aa_4} = H.v$$

as stated above. This is the general principle involved in Culmann's "Summation Polygon." The notation employed above seems to forbid the possibility of a mistake in construction.

On applying this rule to the products $(P_1x_1 + P_2x_2)$ and $(P_3x_3 + P_4x_4)$ separately, beginning the construction for the last at a_2, we see that $(P_1x_1 + P_2x_2) - (P_3x_3 + P_4x_4) = H(aa_2 - a_2a_4)$, a principle that we shall find of great utility.

To apply the above principle to *areas*,

as the one shown on the left, draw through each corner of the plat, lines $\perp P_1 P_4$, thus dividing the figure into triangles and trapezoids. Take the horizontal medial lines x_1, x_2, . . . of each triangle and trapezoid in dividers, and lay off on line xx as above. The lines P_1, P_2, . . . are the altitudes of the triangles or trapezoids; so that by choosing a pole O at a distance H (an even number is best) from $P_1 P_4$, and proceeding as above we find $(P_1x_1 + P_2x_2 + \dots) = H.v$ =area of figure. In this case H and v are measured to the same scale.

If another plat is made alongside of the one shown, the area of both may be found at one operation; the distances x_1, x_2, . . will now denote the sum of the medial lines of the corresponding trapezoids of both figures. Where the same area has a slice taken out of it, this principle will be needed.*

It is generally convenient to choose H about the mean width of the plot, whence v will be the mean height about, and the construction to the right is kept within bounds.

8. Suppose it required to find $(P_1x_1^2 + P_2x_2^2 + \dots)$ or the *moment of inertia* of the forces P_1, P_2, . . . acting at $x_1, x_2 \dots$ about the axis aa. After making the above construction, regard the segments aa_1, a_1a_2, . . . as forces applied at $x_1, x_2, \dots$, and make a construction similar to the above with a pole distance H′, and denote the segments cut off on the line aa_1 by bb_1, b_1b_2, ... Then we have, as before,

$$\overline{aa_1}.\, x^2 = H'.\overline{bb_1},\ \overline{a_1a_2}.x_2 = H'.\overline{b_1b_2}, \text{ etc.}$$

Multiplying both sides by H and adding, noting that by art. 7 $H.\overline{aa_1} = P_1x_1$, etc., we find,

$$(P_1x_1^2 + P_2x_2^2 + \dots) = H'.H.\overline{bb_4}.$$

For a full discussion of the elements of Graphical Statics, the reader is referred to DuBois' *Graphical Statics*, and Eddy's *Researches in Graphical Statics*. We have already nearly all the principles of the method that we shall use in this treatise, and shall now proceed to their application in the case of the arch.

THEORY OF ELASTICITY.

9. Let Fig. 3 represent a portion of a *solid arch with parallel flanges*, whose neutral axis, when ordinary flexure alone is considered, is nn.

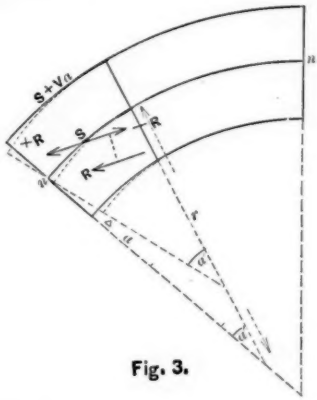

Fig. 3.

Consider the part, whose length measured along nn is s, included between the two normal planes that make an angle a before strain and a' after strain. Let R be the resultant of the external forces at the middle of the part taken. At the middle of s on nn conceive two equal opposed forces +R, −R, each equal to R. The force R is thus equivalent to a couple $R\bar{R}$ and a force +R at the middle of s on nn. This latter force may be decomposed into two components T and N tangential and normal to nn at the point of action. The forces T on each part produce a shortening of the entire arch and its effect may be considered by itself, as we shall see. The force N is similar to the shearing strain in a straight beam (see art. 82). The moment RR is principally effective in changing the curvature of the arch, and it will alone be considered in this connection at present. The deformation caused by N is of course too small to be regarded in the solid arch with a continuous web. See art. 84 for its effect on braced arch. Call the distance of any fiber from nn, v; its length before flexure is $s + va$, after flexure $s + va'$. Denote the modulus of elasticity by E, and the area of the cross sec-

* I have recently used the above method in checking numerical computations, and find that for small tracts of land (30 acres say), the difference between the graphical and numerical methods was nearly zero, for two 100 acre tracks, $\frac{1}{10}$ of an acre difference; for a 250 acre tract, 3.1 acres; for a 400 acre tract 2.2 acres. Scale used was 500 feet = 1 inch. The above is only intended to give some idea of the accuracy of the method, using only ordinary care.

tion of the fiber by A: the force on the fiber is,

$$f=\frac{(a-'a)}{s+va}\mathrm{EA}$$

If r is the mean radius of curvature at the cross section, $ra=s$, which substitute above. Now the sum of the moments of the forces f, above and below nn, must equal the moment of the external forces $\mathrm{R}\overline{\mathrm{R}}=\mathrm{M}$.

$$\therefore \mathrm{M}=\Sigma\frac{v^2(a'-a)}{(r+v)a}\mathrm{EA}$$

The term $\Sigma v^2\mathrm{A}=\mathrm{I}$=moment of inertia of cross section. In taking the above sum, we see that if the cross section is symmetrical with respect to nn, that for equal values of v above and below nn, the sum of the two moments

$$\frac{\mathrm{E}v^2\Delta a\mathrm{A}}{(r+v)a}+\frac{\mathrm{E}v^2\Delta a\mathrm{A}}{(r-v)a}=\frac{2}{ra}\cdot\frac{\mathrm{E}\Delta a v^2\mathrm{A}}{1-\frac{v^2}{r^2}}.$$

When $\frac{v^2}{r^2}$ is very small, as it generally is, it may be neglected, so that regarding E and r as constant for the length s, we have

$$\mathrm{M}=\frac{\Delta a.\mathrm{EI}}{s} \quad . \quad . \quad . \quad (1)$$

$$\therefore \Delta a=\frac{\mathrm{M}.s}{\mathrm{EI}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

10. If we denote by f' the strain per unit of area on the fiber, most distant from nn, whose distance from nn is v', we have from the value of $\frac{f}{\mathrm{A}}$,

$$f'=\frac{v'.\Delta a.\mathrm{E}}{(r+v)\Delta a}=\frac{v'.\Delta a.\mathrm{E}}{s},$$

when v is small compared to r; so that (1) may be written,

$$\mathrm{M}=\frac{f'}{v'}\mathrm{I} \quad . \quad . \quad . \quad . \quad (3)$$

It will be well for the reader to bear carefully in mind the approximations introduced.

We have omitted the deformation due to T, for the present. In the solution given in Du Bois' Graphical Statics, this shortening of the axis, is included from the beginning. Its influence, however, is generally very small. We shall include its effect under the head of temperature strains.

11. It may be observed that in a curved beam the neutral axis does not exactly coincide with the center of gravity of the cross section, since the elongations *per unit of length* of the fibers, is not the same at equal distances either side of the real neutral axis, though very nearly so for flat arches. Assuming nn to coincide with the centers of gravity of the successive cross sections, we find the strain on the outermost fibers due to M and force T as follows:

From the middle of s draw a perpendicular to nn to intersection with R, and call its length p. Now if R is decomposed into components N and T, the former produces no moment about the middle of s; hence $\mathrm{M}=\mathrm{T}p$.

Calling A the area of the cross section and g its radius of gyration, $\mathrm{I}=g^2\mathrm{A}$, and (3) may be written

$$\mathrm{T}p=\frac{f'}{v'}g^2\mathrm{A} \therefore f'=\frac{\mathrm{T}}{\mathrm{A}}\frac{v'p}{g^2}.$$

The force T, at the middle of s, produces a uniform strain on the fibers of the cross section there $=\frac{\mathrm{T}}{\mathrm{A}}$, so that the total strain on the outermost fiber per unit is,

$$f_o=\frac{\mathrm{T}}{\mathrm{A}}\left(1\pm\frac{pv'}{g^2}\right) \quad . \quad . \quad . \quad (4)$$

12. The last two articles are only of service in estimating the maximum strain on any fiber when the position of R is known. We must now deduce principles by which to find the locus of the resultants on all the cross sections.

The angle $(a'-a)=\Delta a$ (Fig. 3) is the *change* of inclination of the tangents at the ends of the arc s due to the moment M producing flexure.

In Fig. 4 let the arc *abc* represent the neutral line nn of Fig. 3. Also let b, whose co-ordinates with c as the origin are x and y, be the center of a portion s of the neutral line. Let Δa for this portion=angle *cbe*. At c draw $ce \perp bc$ and $ed \perp ac$, or the axis of x.

Then from similarity of triangles

$$\frac{cd}{ce}=\frac{y}{bc} \therefore cd=\frac{ce}{bc}y=y.\Delta a$$

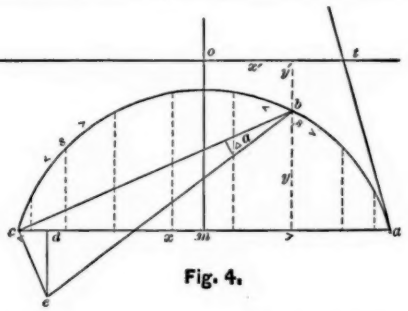

Fig. 4.

$$\frac{de}{ce}=\frac{x}{bc} \therefore de=\frac{ce}{bc}x=x.\Delta a.$$

The angle Δa being very small we have replaced its tangent by the arc itself.

As before observed, the moment M, at the middle of the little arc considered, s, being small, is taken, approximately, as producing the same change Δa, as the real moments acting. Again, the change Δa is made up of the numerous little changes occurring at each little elementary portion of the arc s, so that if each portion was considered as above, x and y would have values above and below the co-ordinates of the middle point b. We assume, therefore, that if M, x and y are taken at the middle point, b of the arc s considered, that the true horizontal and vertical displacements of c are given very nearly by the above equations. Some such approximation is inseparable from the graphical method. Therefore divide up the arc abc into a number of parts, deduce the horizontal and vertical displacements for each portion and find their sums, $\Sigma y.\Delta a$, $\Sigma x.\Delta a$.

If the tangent at a moves through a small angle β, we have de due to it$=x\beta$ $=\overline{ac}.\beta$; the horizontal displacement is $y\beta=o$. So that, calling h and v the total horizontal and vertical displacements of the left end of the arc, we have,

$$h=\Sigma_a^c(y\Delta a) \quad . \quad . \quad . \quad (5)$$

$$v=\Sigma_a^c(x\Delta a)+\beta.\overline{ac} \quad . \quad (6)$$

It is believed that the results attained by measuring x and y to the middle of each little elementary arc are more nearly correct, than when they are measured to the extremities of the little arcs.

The arc may be divided into equal or unequal parts. It can easily be divided into *equal* parts by the dividers: thus, dividing abc into two parts, these parts into two others and so on. The result is evidently more correct the greater the number of divisions.

13. From eq. 2, we find Δa for one elementary arc. By adding the values found for each portion, we have for the total change of the inclination of the tangent at c with respect to the tangent at a

$$\Sigma(\Delta a)=\Sigma_a^c\frac{Ms}{EI} \quad . \quad . \quad . \quad (7)$$

Substituting the value of Δa from (2) in (5) and (6), we have,

$$h=\Sigma_a^c\frac{Mys}{EI} \quad . \quad . \quad . \quad (8)$$

$$v=\Sigma_a^c\frac{Mxs}{EI}+\beta.\overline{ac} \quad . \quad . \quad (9)$$

If we divide the neutral line abc into equal parts s, as suggested, s may be placed before the sign of summation.

We must now find M, E, I, x and y, for the middle of each part and sever the separate displacements for each part between the limits a and c, as in fact, the equations indicate.

We shall generally, hereafter, for simplicity, suppose E and I constant, so that they may be placed before the sign of summation.

It may be well to state here, that if the actual moments M, acting on each division of the arch, are known, together with E and I, that we can find from eqs. (8) and (9) the horizontal and vertical displacements of each point of the arch with reference to a, by regarding the

point c, the origin of co-ordinates, at each point in turn, and effecting the summation from a to c. We can thus compute, and lay down on a drawing, the exact form assumed by the neutral line after strain. By this means we may predict the lowering of the crown or spreading at the haunches, which last in the stone arch, is generally resisted partially by the spandrels. This partial resistance, however, causes a new curve of pressures, as we shall see, thus changing the values of M, so that it does not seem possible by this means to compute the actual resistances of the spandrels.

14. From the last three equations we obtain the conditions necessary to locate the positions of the resultants of the external forces at every cross section. Thus, *for an arch whose tangents at the abutments are fixed in direction, span invariable and vertical deflection of end zero, i.e.*, for an *arch fixed at the ends*, then eqs. (7),(8) and (9), and β, are each equal to zero.

$$\therefore \ \Sigma_a^c \mathrm{M} = o, \ \Sigma_a^c \mathrm{M}y = o, \ \Sigma_a^c \mathrm{M}x = o$$

15. *If the arch is fixed in direction at a, but hinged at c*, so that the tangent there may change, we have $h=o$, $v=o$, $p=o$,

$$\therefore \ \Sigma_a^c \mathrm{M}y = o \ \ \Sigma_a^c \mathrm{M}x = o$$

the origin of co-ordinates being at the hinged end.

16. *If the arch is hinged both at a and c*, the span being invariable,

$$\Sigma_a^c \mathrm{M}y = o$$

It is true that $v=o$ in eq. 9, but since β now has some value, it follows that $\Sigma \mathrm{M}x$ cannot be zero as before.

17. *Lastly, if the arch is fixed in direction at the ends, and hinged at some intermediate point b*, the horizontal and vertical displacements of b, for the part ab, must be the same as for the part bc, owing to the connection. So that we have the conditions,

$$\Sigma_a^b \mathrm{M}x = \Sigma_c^b \mathrm{M}x, \quad \Sigma_a^b \mathrm{M}y = -\Sigma_c^b \mathrm{M}y.$$

The last term is minus, since if $\Sigma \mathrm{M}y$ on one side is plus, on the other side it is minus, to cause the horizontal displacement for the two parts to be in opposite directions. From the reasoning of art. 12, we see that the origin of co-ordinates for this case must be taken at b, y vertical, x horizontal as before.

18. For the arch "fixed at the ends" we may take the origin of co-ordinates at some other point than c, as o, if preferred.

For we have, calling $\overline{cm}=d$, and $\overline{om}=d$, the old co-ordinates of b, x and y, the new, x' and y': $x=d-x'$, $y=d'-y'$, so that (art. 14)

$$\Sigma \mathrm{M}x = \Sigma \mathrm{M}(d-x') = d\Sigma \mathrm{M} - \Sigma \mathrm{M}x' = o$$
$$\Sigma \mathrm{M}y = d'\Sigma \mathrm{M} - \Sigma \mathrm{M}y' = o$$
$$\therefore \ \Sigma \mathrm{M}x' = o, \ \Sigma \mathrm{M}y' = o.$$

Since by the first condition $\Sigma \mathrm{M} = o$. It is important to observe that x' must change sign on the other side of om in the summation. Similarly for y' if o is below the crown.

If we are using a trial curve of pressures for which $\Sigma \mathrm{M}$ is not zero, we must of course reckon the origin at c to find the real sums, which may not be zero if the curve is not located properly.

Similarly for an arch, with one or more hinges, the origin must be at the hinged point, since $\Sigma \mathrm{M}$ is not zero, &c.

19. If in any case the arch is not hinged on the neutral line abc, but at some point outside of it, this point must be taken as the origin of co-ordinates, since for this point the condition that the horizontal and vertical displacements equal zero is alone true, as follows from art. 12. If the ends of the arch are flat against the skewbacks, but not bolted to them, the bending moment at the abutment may become large enough to lift one edge clear of the abutment, which presents a case similar to the above. It is plain that we divide up the *neutral line* abc into equal parts as before, and that we must now attend to the signs of x and y, if any of the values are minus. This case has been treated by Mr. Charles Pfeifer, C.E. (V. N.'s Mag. for June, 1876), though the use of his eqs. (17) and (18) involve the principle that the deflections of the end of the centre line are zero; whereas the true conditions are that the deflections of the hinged, or free-to-turn point are zero. The other view would involve our taking x and y about c in Fig. 4, in place of the point about which the arch rotates.

This error though is generally very small. In DuBois' *Graphical Statics*, p. 388, art. 22, is given an example of "Arch continuous at crown and hinged at ends of lower rib," the formulae deduced for arches hinged in the neutral line at the ends being supposed to apply, which is evidently incorrect.

20. When the moment of inertia and modulus of elasticity vary for different parts of the arch, we have only to replace M in arts. 14 *et seq.* by $\frac{M}{EI}$, when the above conditions hold. In designing an arch, it is of course impossible to know the values of I beforehand, so that some section must be assumed by which to compute a more correct one, and this in turn assumed, and so on; by this means approximating to the true result.

21. In Fig. 5, let the force P be in equilibrium with the forces V_1 and F at c_1

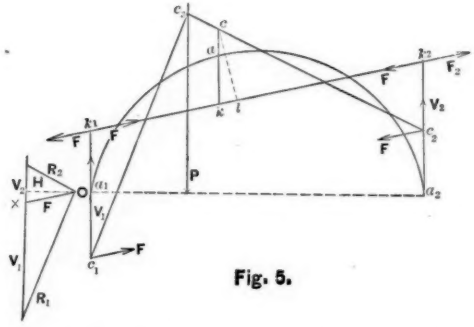

Fig. 5.

and V_2 and F at c_2, the forces F being parallel. The resultant of V_1 and F at c_1 must intersect the resultant of V_2 an F at c_2, in some point c_3 of P. Lay off P on the load line on the left, draw R_1 and R_2 parallel respectively to c_1c_2 and c_2c_3 to intersection o. Then R_1=resultant of V_1 and F at c_1, and R_2=resultant of V_2 and F at c_2. Hence, drawing the line F through o, parallel to force F, we see that the forces at c_1 and c_2 must have the intensities shown by the sides of the triangles of the force diagram on the left. Now draw a line k_1k_2 parallel to the direction of force F, also draw the lines c_1k_1 and c_2k_2, parallel to P. It will not affect equilibrium to add two equal opposed forces F at k_1, also at k_2 acting parallel to F, all of them; but since the two forces that act towards each other now balance, the effect is to form a couple at the left with the F at c_1 and the remaining F at k_1, similarly at the right. If we denote the angle made by F with a line perpendicular to P by a, we have the moment of the couple at c_1=F. $k_1\overline{c_1}$. cos a=H. k_1c_1 and at c_2, F.$k_2\overline{c_2}$. cos a=H.$k_2\overline{c_2}$, H denoting the pole-distance of force polygon, as shown by the dotted line from o.

Now since our system of forces is balanced at each apex, k_1, c_1, c_2, c_3, k_2, on supposing opposed forces R_1, R_1 acting along c_1c_2, and R_2, R_2 along c_2c_3 as we see by reference to the force polygon, if we suppose the forces removed to the right of the same line ck, eqilibrium still holds at c_3, c_1 and k_1, so that the sum of the moments of the forces there about any point c must be zero. Suppose the two opposed forces R_1, acting along c_1c_2, removed, we have left P, V_1, couple FF, force F acting to the right at k_1, and force R_2 on cc_2. The moment of this last is zero; hence, denoting the lever arms of V_1 and P about c by x and d, we have

$$(V_1x-H.\overline{c_1k_1}-Pd)=F.\overline{cl},$$

$\overline{cl}$ being perpendicular to k_1k_2. Draw kc parallel to P; then from similarity of a triangle in force diagram to ckl, we have F.$\overline{cl}$=H.$\overline{kc}$. Now if we had a structure, as a continuous girder acted on by V_1, couple FF and P to the left, we should call the term $(V_1x-H.\overline{c_1k_1}-Pd)=M_c$, the moment of the external forces left of the section taken; whence,

$$M_c=H.\overline{kc}$$

This same result holds to the left of P, the term Pd being then omitted. If c is taken below the closing line k_1k_2, as we shall call it, the moment of F is of an opposite kind to the first, so that M_c changes sign.

We see from the foregoing that $k_1\,c_1\,c_2\,c_2\,k_2$ is the equilibrium polygon for the forces considered, and that the moment M_c at any point is equal to the pole distance H multiplied by the ordinate $\overline{kc}$ of the equilibrium polygon to the closing line. The effect of an end moment is thus to shift the closing line at the end an amount equal to the moment divided by H. If the forces of the end couple as given, do not coincide with F and F, we can replace it by another couple of same moment whose forces do coincide with F and F as drawn.

If there are a number of forces parallel to P, the equilibrium polygon c is formed as usual, and the above relation evidently holds.

22. Let us now conceive a_1aa_2 (Fig. 5) to be the neutral line of an arch rib fixed at the ends and acted on by a single weight P, the real reactions being R_1 and R_2, acting at c_1 and c_2, respectively, in the directions c_1c_3 and c_2c_3. If we decompose each reaction into horizontal and vertical components, we see that the former alone produce the end moments which cause fixity; these moments being $H.\overline{a_1c_1}$ and $H.\overline{a_2c_2}$ respectively.

In case that the arch is hinged at the end, there is no moment, so that the reaction must then pass through a_1, a_2, or both, according as the arch is hinged simply at a_1 or at a_2, or at both of these points. Now decompose R_1 and R_2 at c_1 and c_2 into components v_1, F and v_2, F.

As before, let us apply along some line k_1k_2, parallel to the direction of F, four forces F acting away from each other at k_1 and k_2 as shown in the figure. This does not disturb equilibrium, but it has the effect of transferring F at c_1 and c_2 to k_1 and k_2, and of adding the couplets $H.\overline{k_1c_1}$ and $H.\overline{k_2c_2}$ at the left and right abutments respectively.

Taking moments about some point a of the arch of the forces, as now disposed, to the left of a, we have,

$$M=(Vx-H.\overline{k_1c_1}-Pd)-H.\overline{ak};$$

noting that the moment of F at k_1 about $a=H.\overline{ak}$. Now we see that the term $(Vx-H.\overline{k_1c_1}-Pd)$ *is the moment at a of a girder acted upon by the force* P, *with end moments*, $H.\overline{k_1c_1}$ and $H.\overline{k_2c_2}$ *at* a_1 *and* a_2 *respectively, the pole distance* H *being the same as the actual horizontal thrust of the arch; and that* $k_1c_1c_2c_2k_2$ *is the equilibrium polygon for such a girder, &c., so that the term above,* $M_c=H.\overline{kc}$. Hence we find the bending moment at any point a of the arch to be,

$$M=H(\overline{ck}-\overline{ak})=H.\overline{ac}.$$

Hence *the actual moment at any point of an arch is equal to the horizontal thrust multiplied by the vertical distance from the equilibrium polygon c to the neutral line of the arch,* M *being* $+$ *or* $-$, *as* $\overline{ck}>\overline{ak}$, *or* $<ak$. This curve $c_1c_2c_2$ is the locus of the real resultants acting at points of the arch, and is called *the pressure curve.* It is evidently irrespective of the particular position of k_1k_2, and hence of the end moments $H.\overline{k_1c_1}$ $H.\overline{k_2c_2}$.

Remark.— This result is easily proved otherwise. In fig. 3, from the middle point of arc s, draw vertical and horizontal lines to intersection with R and denote their lengths by v and h; also draw a perpendicular line to R, and call its length p. If the hypothenuse of the large right triangle formed represents R, the side h will represent H. From similarity of triangles we have.

$$\frac{R}{H}=\frac{v}{p} \quad \therefore \; Rp=Hv, \text{ as stated,}$$

23. Now if we regard the neutral line of the arch itself as an equilibrium polygon, with a closing line k_1k_2, due to some kind of loading not given, having a pole distance H equal to the actual horizontal thrust of the arch, the term $H.\overline{ak}$ in the previous equation is the moment of the supposed external forces at a and may be denoted by M_a, so that the previous equation may be written,

$$M=M_c-M_a \quad . \quad . \quad . \quad . \quad (10)$$

This equation holds on the supposition that c_1 and c_2 are the real points of action of the resultants at left and right abutments; but there are no restrictions as to the position of k_1k_2 whatsoever. We shall presently see the necessity of imposing some. Thus for the arch *fixed at the ends,* we have (art. 14)

$$\Sigma M=o,\ \Sigma Mx=o,\ \Sigma My=o \quad . \quad . \quad . \quad . \quad (11)$$

Now if we suppose the end moments, $H.\overline{k_1c_1}$ and $H.\overline{k_2c_2}$ of sufficient intensity to fix in direction the tangents to the neutral line of a girder, acted upon by the same loading as the arch; and if the vertical deflection of one end above the other is zero, we have, since M_c is the moment for such a girder,

$$\Sigma M_c = o \quad \Sigma M_c x = o \quad . \quad . \quad . \quad . \quad (12)$$

If we subtract the first two of eqs. (11) from eqs. (12), we have as a necessary consequence from eq. (10),

$$\Sigma M_a = o \quad \Sigma M_a x = o \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Now we shall find that by eqs. (12), we were enabled to find the end moments $H.\overline{k_1c_1}$, $H.\overline{k_2c_2}$, without knowing H, or the position of the closing line. We shall further find that eqs. (13) enable us to find this closing line, k_1k_2, in position. Lastly from the condition,

$$\Sigma My = \Sigma(M_c - M_a)y = o,$$

we will be enabled to find H, whence from the found end moments, the distances k_1c_1, k_2c_2, may be ascertained and laid off from k_1 and k_2, thus giving us two points of the pressure curve, whence it may be drawn, with the known value of H.

24. It is true that we may determine some other closing line by assuming,

$$\Sigma M_c = A, \quad \Sigma M_c x = B$$

whence by subtraction, as before, we have as a consequence,

$$\Sigma M_a = A, \quad \Sigma M_a x = B$$

But it is apparent since the total bending is now proportional to A (see art. 13) that the vertical deflection is no longer at zero, but dependent on the value assumed for A, so that B is a function of A and cannot be assumed arbitrarily. But as this involves a computation of B, the steps are not near so simple, as when we assume the total bending of the supposed girder zero, when of course the vertical deflection is so likewise.

25. From similar considerations of simplicity, it will be assumed that when an arch is hinged at any point or points, that the supposed girder is similarly hinged, so that the closing line k_1k_2 will pass through the point or points (not exceeding two) since the moment there is zero.

The solution corresponding to Fig. 5 comprehends every case, the closing line changing its position according to the conditions of the case.

Thus for the cases treated in arts. 14–17, the formulae there given for the *arch* hold.

It is then evident that whichever of the similar conditions apply to the *girder*, apply likewise to *curve a* regarded as an equilibrium polygon as follows from the relations expressed by eq. 10.

26. We have hitherto, for simplicity, considered but one weight P on the arch; but since for any number of weights the two straight lines c_1c' and $c'c_2$ are simply changed into a polygon cc, the equilibrium polygon (art. 2) due to the weights alone, the result holds for any loading whatsoever. The above demonstration is essentially the same, with some emendations, as that given by the writer in VAN NOSTRAND'S MAGAZINE for June, 1878.

27. The above principle was first stated by Prof. H. T. Eddy in the January, 1877, No. of VAN NOSTRAND'S MAGAZINE, in these words:

"If in any arch that equilibrium polygon (due to the weights) be constructed which has the same horizontal thrust as the arch actually exerts; and if its closing line be drawn from consideration of the conditions imposed by the supports, etc.; and if, furthermore, the curve of the arch itself be regarded as another equilibrium polygon due to some system of loading not given, and its closing line be also found from the same considerations respecting supports, etc.; then when these two polygons are placed so that these closing lines coincide and their areas partially cover each other, the ordinates intercepted between these two polygons are proportional to the real bending moments acting in the arch."

As Prof. Eddy claims, this proposition supplies the hitherto missing link in the graphical treatment of solid or braced arches.

In art. 34, see an equally general method, which does not involve a demonstration of this proposition, though identical with it in the construction.

ARCH WITH FIXED ENDS.

28. Let the curve aa_4a (Fig. 6) which

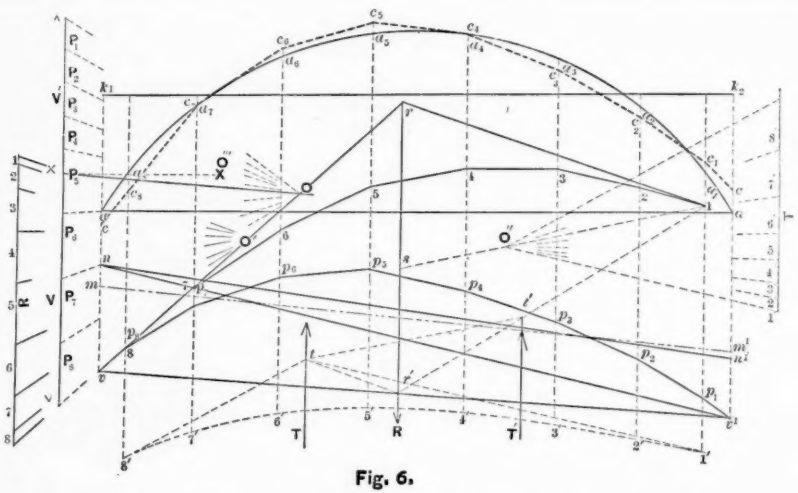

Fig. 6.

may be of any kind, be the neutral line of an arch rib. Divide it into equal parts (8 in the figure), and at the middle of these parts a_1, a_2, . . . draw vertical ordinates y to the axis of x, aa. The origin may be taken at either end of the span in this case.

Conceive the vertical loads to be applied at the points a_1, a_2, . . . , of the intensities P_1, P_2, . . .

Lay off these loads to scale on the load line P_1P_8, choose a pole o, and beginning at some point v in the vertical through a draw $vp_8 \parallel$ ray VP_8 (it coincides with it in the Fig.) then $p_8p_7 \parallel$ ray P_7P_8, etc., to intersections with the verticals through a_8, a_7, etc. From the last point v', so found, draw a closingl ine vv'; a line through o parallel to it would give the reactions of the arch acting as a girder with no end moments, since vp_5v' is the equilibrium polygon for the forces (art. 2). It will conduce to brevity to designate the curves or polygons that are marked with the same letter, though different subscripts, by that letter; thus, the curve $aa_1a_2 \ldots a_8a$ will be called the curve a; similarly we have the polygons p and c.

In order to find the true closing line for the arch acting as a girder, we have the conditions, art. 23,

$$\Sigma M_c = o, \quad \Sigma M_c x = o.$$

Let nn' be a trial closing line, then since the bending moments M_c are proportional to the ordinates of the equilibrium polygon, $nvpv'n'$, the first condition requires, that the sum of the vertical ordinates through p_1, . . , p_8 from nn' downwards to pp=sum of ordinates above nn' to polygon pp. It is easily seen from the considerations of art. 4, that the bending moments are of different sign when the ordinates lie on opposite sides of the closing line.

If we add to the above equality, the sum of the ordinates intercepted between vv', the line nn', and the part of the polygon p below nn, we have the condition that the sum of the ordinates included between vv' and nn' shall equal the sum of the ordinates from vv' to polygon pp; hence if we denote the ordinates from vv' to pp and nn' respectively by p and n, and the ordinates between nn' and the curve pp by m, we have,

$$m = p - n$$

The second condition, $\Sigma M_c x = o$, may be written $\Sigma mx = \Sigma px - \Sigma nx = o$; which indicates that if the ordinates p and n are regarded as forces, that the sum of their moments about an abutment are equal; *i.e.*, the resultant of the p's coincides with the resultant of the n's, since $\Sigma p = \Sigma n$, from the conditions $\Sigma m = o$, as proved above.

Therefore find the resultant R of the

ordinates p regarded as forces, in position, direction and magnitude. This may be done by calculation, or the graphical method given in art. 3 may be more conveniently employed. In the latter case we have only to lay off the ordinates p ($\frac{1}{2}p$ was taken as more convenient) on a vertical line 18 precisely as we did $P_1, P_2, \ldots$, choose a pole o', then starting at p_8 say, draw the equilibrium polygon 87654321 as usual. On prolonging the first and last sides, corresponding to vp_8 and $v'p_1$ of the first polygon, to intersection r, a vertical through this point will give the position and direction of R. In magnitude, $R=\Sigma p$, as taken from the force line.

29. Now draw a line nv' dividing the ordinates n into two sets. Lay off the half of the ordinates (through each point of division $a_1, a_2, \ldots$) intercepted between vv' and nv', as forces on the line $8'\ldots1'$, and similarly find their resultant T in position. Now the position of T is not changed when nn' is changed, whence nv' is revolved about v' through some angle: for all of the ordinates in the triangular portion $vv'n$ are altered in the same ratio, as is shown in *Geometry* (see Chauvenet's Bk. III, Prop. 8).

It follows that T', the resultant of the half ordinates included between nv' and nn', is at the same distance from v' that T is from v.

The problem then is this, having R in position, &c., and T and T' in position only, what are the real magnitudes of T and T'?

Laying off $\overline{rr'}=R$, draw from some point $\overline{1r}$ and $\overline{1r'}$. Next through some point r' on R draw $\overline{r't}$ and $r't'$ to t and t'; a line $\overline{1s}$, drawn parallel to the closing line tt', divides $\overline{rr'}$ into segments rs and sr' equal to T and T' respectively. Now if the load line $\overline{8'1'}$, which represents the trial T, is not equal to $\overline{rs}$, we must reduce the length vn to vm in the ratio of $\overline{8'1'}$ to $\overline{rs}$, for then each ordinate in the triangle $vv'n$ is reduced in this ratio, so that their half sum will now equal $\overline{rs}$. Similarly the half sum of the ordinates in the triangle $vv'n'$ (which can be taken by dividers) should equal $\overline{r's}$, if n' has been accurately located. If not change $\overline{v'n'}$ to $\overline{v'm'}$ in ratio of $\overline{rs}$ to the half sum mentioned, when all the ordinates in triangle $nv'n'$ will be altered in this ratio, and hence their sum. Now the sum of similar ordinates in the triangle $nv'm'$, is the same as for the triangle $mv'm'$, the lengths being the same in either case; whence mm' is the true closing line to satisfy the two conditions $\Sigma M_c=o$ and $\Sigma M_c x=o$. It is well to test the first, by adding the ordinates to mm' from curve p, and see whether the sum of those above mm' is equal to the sum of those below the same line.

30. It is well also to choose the poles o' and o'' so that the exterior rays will form angles of nearly 90° with each other, as then the positions of r and t can be determined more accurately. The ordinates were halved for the force lines simply to economize space. When the loading is symmetrical with respect to the crown (as for a uniform load, for example) the ordinates of the polygon p are symmetrical about the center vertical, hence the closing line mm' is drawn parallel to vv', and is quickly fixed by the condition $\Sigma M_c=o$ as before. The other condition $\Sigma M_c x=o$ holds also; in fact the requirement that mm' be parallel to vv' is a consequence of this condition.

31. Let us next fix the closing line k_1k_2 of the curve a, regarded as an equilibrium polygon, by the conditions, $\Sigma M_a=o$, $\Sigma M_a x=o$.

The resultant of the ordinates from the span line aa to curve a passes through the crown in consequence of the symmetry of curve a about the crown, also the resultant of the ordinates from span aa to k_1k_2 (see art. 28), since then the condition $\Sigma M_a x=o$ is satisfied. But the latter resultant cannot pass through the crown unless k_1k_2 is horizontal. Therefore draw a trial kk; add, by the dividers, the ordinates below it to curve a, also those above it; their difference (if any) divided by the number of ordinates (8 in this case) will give the amount to raise or lower $\overline{kk}$ so that the conditions $\Sigma M_a=o$, is satisfied.

32. We have now the condition

$$\Sigma My=\Sigma(M_c-M_a)y=o$$

as the last one by which to locate the curve c in its true position, and determine the true horizontal thrust of the arch.

From the last equation we have

$$\Sigma(M_c y)=\Sigma(M_a y)$$

Now M_c is proportional to the ordinates of curve p from its closing line mm'; those above it being regarded as plus, say those below it minus. Similarly M_a is proportional to the ordinates from $\overline{kk}$ to curve a; both curves being supposed to have the same pole-distance. Hence we sum the above products, and if the equality does not hold, we must change the pole-distance of curve p (which was assumed arbitrarily) until it does subsist. We can get the above sums, and their ratio, by calculation, by scaling off the ordinates, multiplying y by its corresponding M, &c., but the graphical method is here very direct and even accurate.

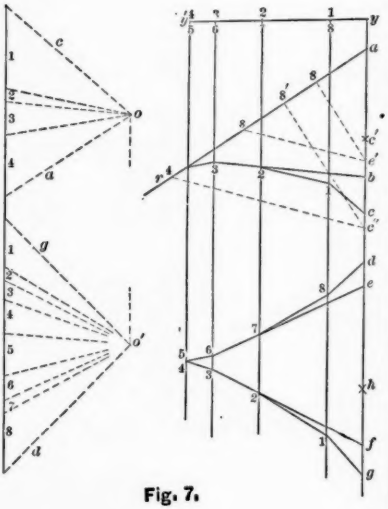

Fig. 7.

To employ it, for the object in view, draw a line yy' (Fig. 7) $\perp yg$, and lay off from y the distances, $y1$, $y2$, etc., equal to the ordinates y at a_1, a_2, etc., and draw lines $\parallel yg$ through 1, 2, etc.

Next on a line $\parallel yg$ lay off in order the ordinates from kk to curve a. We have only laid off the ordinates at a_1, a_2, a_3, a_4, since the sum $\Sigma M_a y$ is the same on the other side of the crown. Then choosing a pole o, and starting at some point c on yg, draw $c1 \parallel$ ray $c1$, $12 \parallel$ ray 12, . . . $4a \parallel$ ray $4a$ to intersection a with yg; also produce 32 to intersection b with yg; then art. 7,

$$\Sigma M_a y = 2H'x(ab - bc) = 2H'.\overline{ac'},$$

on making $bc' = bc$.

Again, lay off on a line $\parallel yg$, in order, the ordinates proportional to M_c, intercepted between mm' and polygon p. With the same pole distance H' choose a pole o'; then starting at g on yg draw $g1 \parallel$ ray $g1$, etc., and produce the sides 32 and 67 to f and e, respectively. Then we have,

$$\Sigma M_c y = H'x[ef - (de + fg)] = H'.\overline{eh},$$

if $\overline{fh} = de + fg$, since $H'.\overline{ef}$, by art. 7, is the sum of the products $M_c y$ for the ordinates above kk, and $H'(de + fg)$; a similar sum for these below. Hence laying off $\overline{ac''} = 2\overline{ac'}$ and $\overline{ae'} = \overline{eh}$, since they are not equal, we must diminish the pole distance of polygon p in the ratio $\frac{ae'}{ac''}$, which thus has the effect of increasing the ordinates from mm' to polygon p in ratio $\frac{ac''}{ae'}$, in order that the equality above may hold. Thus, to find the true pole distance, we lay off the assumed pole distance from o to the line P_1P_8 of figure on some line as $\overline{ar}$ figure from a to r. On drawing $e's \parallel c''r$, the distance $\overline{as}$ is the true pole distance. Hence draw a horizontal line, through the point of intersection of the line drawn through $o \parallel mm'$ with P_1P_8 (Fig. 6), a distance to o''' equal to as (Fig. 7). The reactions v and v' are the same as before, and since our actual closing line k_1k_2 in position is horizontal, the left figure is the true force polygon corresponding to the $M_c's$. Now by art. 4, we have to elongate the ordinates from polygon p to mm in ratio $\frac{ac''}{ae'}$. Thus lay off the ordinate through p_8 (Fig. 6), from a to 8 (Fig. 7), draw $c'8' \parallel e'8$ whence $a8'$ is the true length of this ordinate. Lay it off from line k on 8th ordinate downwards to c_8.

Similarly for the other ordinates of polygon p, so that the polygon $cc_1c_2 \ldots c_8c$ may be located; or, if preferred, having found one point as c_5 in this manner, polygon c may be drawn by means of the force polygon $O''' - P$.

The extremities c and c thus found, as well as other points, should agree with those found by the first method.

33. Now it is evident that the polygon

c of Fig. 6, is identical in meaning with polygon c of Fig. 5, as well as the closing line; so that calling the pole-distance from O‴, H, we have the bending moment at any point a_5 of the arch $=H.\overline{a_5c_5}$, etc.

This is plain, because polygon c of Fig. 6 satisfies the conditions $\Sigma M_c=o$, $\Sigma M_c x=o$, and polygon a the conditions $\Sigma M_a=o$, $\Sigma M_a x=o$; whence by subtraction in connection with eq. 10 (art. 22), we have $\Sigma M=o$ and $\Sigma Mx=o$.

Again from the last condition above, we have, $\Sigma M_c y-\Sigma M_a y=\Sigma My=o$. So that all the conditions given in art. 23, of an arch fixed at the ends, are fulfilled.

It is well to test the ordinates of the type ac, to see if the three conditions are fulfilled. Thus the sum of the $\overline{ac}$'s above curve a, should equal the sum of those below. The sum of the products Mx or $\overline{ac}.x$ above and below curve a should be equal; similarly for $\overline{ac}.y$, &c. The multiplications can be easily effected by the method of art. 32.

Now cc is the true *pressure curve* of the arch; *i.e.*, the reaction at the left abutment is at c, and equals the lower ray (VP_5) through O″, its componenents being V and H; similarly the resultant (=ray $V'P_1$ from O‴) at right abutment acts at c, its components being V′ and H; so that the moments at these points are, $H.\overline{ac}$, the distance ac being different at the two abutments for an unsymmetrical load. Moreover we see that the curve c is found by combining the resultant at c, on the left, say with the force P_5, acting through the vertical at a_5; this resultant with the force at a_4, etc.; so that the rays of the force polygon O‴P represent the resultants in magnitude at the middle (nearly) of the corresponding divisions. Thus, ray $O'''-P_5P_5$, is the magnitude of the resultant, acting along c_5c_5, for the middle of arc a_5a_5, &c.

On decomposing this resultant into components, parallel and perpendicular to chord a_5a_5, we have the tangential and normal components T and N of the thrust acting at, or very near, the middle of the division a_5a_5; whence by art. 11, the greatest stress on any fiber may be found. Similarly for the other divisions of the arc. It may be observed that in place of (Tp) in eq. 4 (art. 11) we may substitute the corresponding (H.$\overline{ac}$) the $\overline{ac}$ being taken at the middle of the portion of the arc considered. We take the "middle," if the loading is continuous, for the resultants given pertain to that point of the arc where the load is supposed divided, as we see by analogy to the previous treatment of the Voussoir arch.

34. We shall now give a second general method of drawing the pressure curve c, fig. 6, that involves only the propositions of Graphical Statics given in arts. 2 3, 4 and 5 of this paper, with the remark of art. 22. Let c fig. 6, be the actual pressure curve, drawn with the force polygon $O'''P_1P_5$; thus satisfying the conditions, $\Sigma M=o$, $\Sigma Mx=o$, $\Sigma My=o$, for the arch fixed at the ends. Now if we draw a trial pressure curve p (which may be supposed to occupy some such position about the arch as curve c, though it is drawn below for the sake of clearness of diagram) with an assumed pole O, we know that if this pole is afterwards changed to O‴, that the ordinates of polygon p are altered in the ratio of old to new pole distance. This alteration, we shall see, is determined entirely from the condition $\Sigma My=o$. Now M being proportional to the ordinates of the type $\overline{ac}$ (art. 22, remark) if we denote the ordinates from some line as k_1k_2 to curves a and c by $\overline{ka}$ and $\overline{kc}$ respectively, we have $\overline{ac}=\overline{kc}-\overline{ka}$.

It is seen that $\overline{kc}$ and $\overline{ka}$ are minus when laid off below kk, since then $\overline{ac}$ is plus, when c is above a, from the preceding equation, and minus otherwise; which must be so, the moments having different signs when c is on opposite sides of curve a.

We have therefore $\Sigma\overline{ac}=\Sigma(\overline{kc}-\overline{ka})=o$ and $\Sigma(\overline{ac}.x)=\Sigma(kc-ka)x-o$. Now if for simplicity we draw kk, as in art. 31, so that $\Sigma\overline{ka}=o$, and $\Sigma(\overline{ka}.x)=o$, it follows that $\Sigma\overline{kc}=o$, $\Sigma(\overline{kc}.x)=o$, also. Now polygon p is simply polygon c out of position and with ordinates all altered in the same ratio; so that we must determine a line mm' that will satisfy the conditions, $\Sigma\overline{mp}=o$, $\Sigma(\overline{mp}.x)=o$, which is readily done, as shown in arts. 28 and 29. As explained above, ordinates on opposite sides of mm' or kk have opposite signs.

Lastly from $\Sigma My=o$, we have $\Sigma(ac.\overline{y}) = \Sigma(kc-ka)y=o$.

$\therefore\ \Sigma(\overline{kc}.y)=\Sigma(\overline{ka}.y)$ or $\Sigma(\overline{mp}.y)=\Sigma(\overline{ka}.y)$

from which condition curve c is drawn as explained in art. 32, the alteration of ordinates not affecting the conditions $\Sigma M=o$, $\Sigma Mx=o$.

On drawing rays P_1P_2, P_7P_8 parallel to $\overline{c_1c_2}$, c_7c_8, thus found, we find O''', the new pole, or better, the old rays, P_1P_2 can have their inclinations altered in the same ratio as the lines p_1p_2, etc. This very general method applies to arches with hinges also, as well as the fixed arch.

ON A NEW PHENOMENON OF STATIC ELECTRICITY.

By G. GOVI.

From Abstracts of Papers published by Institution of Civil Engineers.

The Author refers to previous experiments by M. Duter, in which he also operated with Leyden jars containing liquid. Variations of volume of this liquid were observed by displacement of the liquid in a capillary tube communicating with the interior of the bottle. Disturbing influences of temperature were avoided by an envelope of ice. The Author was led to undertake these experiments by the well-known phenomenon of the piercing of the glass sides of jars too highly charged. The first trials were made with water, and it was found that the water sank in the capillary tube during charging, and regained its level at the moment of discharge, independently of the kind of electricity of the charge. This result appeared favorable to the idea that the sides of the vessel dilated under electric action; but, replacing water by other liquids, it was found that nitric acid contracted, with equal charges, slightly more than water, alcohol much more, and ether behaved otherwise, commencing indefinitely to rise or descend in the capillary tube at the moment of charge, returning at discharge to a higher than its initial level. Olive oil, being non-conducting, gave no appreciable effect. Evidently, neither simple deformation of the receiver, nor difference in conductivity of the liquids, suffice to explain these anomalies. Mercury, under the same circumstances, appears not to contract. This experiment, irreconcilable with variation of capacity of dielectric, led the Author to suppose that the contractions might be explained by admitting that the electrified conducting liquids are condensed against the sides of the vessel. It is generally admitted that the charge of condensers accumulates entirely on the opposed faces of the dielectric, but the armatures of the Franklin square are known to adhere strongly to the glass, and that after removal these always contain sensible quantities of electricity. It is also observed that when a feeble charge is given to a condenser, the armatures alone retain it, the interposed dielectric showing nearly no traces.

The conducting plates of a condenser should and do attract each other across the separating non-conducting matter. Hence the contractions observed may be due to a true condensation (increase of density) of the liquid near the sides of the vessel. Nitric acid being a better conductor than water, if not more compressible, should condense more; and alchohol, a worse conductor, but more compressible, should contract more than the other liquids. Ether, so highly compressible and dilatable, brings into play another phenomenon, a considerable development of heat and expansion of the heated strata which masks the contraction, and gives rise to a true expansion of the liquid. The extremely low compressibility of mercury accords with the absence of contraction observed with this metal. The Author proposes to test his conclusion by substituting for the liquid conductors an insulating liquid, as olive oil, contained in a bottle the interior of which is silvered, and can be put into communication with earth or an electric source. Under these conditions the movements of the liquid would no longer depend on the modifications of the receiver. A Leyden jar, armed only on the interior, and suspended from the arm of a balance in a liquid conductor, which serves as external armature, should increase in weight during charge, if the liquid strata are really condensed against its sides.

A WOODEN HOWE TRUSS BRIDGE.

By JOHN W. HILL, M. E.

Written for VAN NOSTRAND'S MAGAZINE.

SEVERAL months since the writer had occasion to estimate the strength and strain on a small Wooden Howe truss bridge on the line of one of the railways running out of Cincinnati, and believing that the data obtained will be of interest to engineers the following extract from the report to the company is therefore given. The extraordinary lack of harmony and low factor of safety in the several members of the truss exhibit a deplorable and dangerous lack of knowledge upon the part of the designer. At the same time all the bridges on the line of the road are like this in the manner in which they were built, *i.e.* by rule of thumb; and if any are better, it is due to good luck and not intention. The dimensions were obtained from the master bridge builder of the road, and are no doubt correct.

The bridge has been in use for several years, and no question of its ability to carry the usual loads was raised, until the engineer of another company using the bridge had his attention called to its apparent weakness, when he, emphatically (and with good reason) pronounced against its further use until strengthened or rebuilt. The company owning the bridge then set several bents below and shored it up as well as possible, whilst waiting on the new bridge of iron to take its place. The only extenuating circumstance in this case is that the bridge has not yielded to the loads that have come upon it, but this is poor reason for the continuance in use of perhaps fifty other bridges in all probability no better than this one on the line of the road.

The bridge is a *wooden Howe truss*, single span of 88′ divided into *eight* panels of 11′ each. The rise from top of upper chord to bottom of lower chord is 22′, or a real rise measured to centers of chords of 21′.

The lower chord of the truss consists of *four* parallel lines of timber, *two* lines of stuff $6.5'' \times 13''$, and *two* lines of stuff $5.5'' \times 13$.

The upper chord consists of *four* parallel lines of timber, *two* lines $6.5'' \times 12''$, and *two* lines $5.5'' \times 12''$.

The main braces are in pairs each of stuff $8'' \times 10''$.

The counter braces are single each of stuff $7.5'' \times 7.5''$.

The tie rods are in pairs and of these dimensions:

First panel 2″ diam. Second panel 1.75″ diam. Third panel 1.625″ diam. Fourth panel 1.5″ diam. The rods are threaded without upsetting the ends. The floor beams are of stuff $6'' \times 14''$, 21′ long spaced 2.6″ cen to cen. The stringers are each of stuff $10'' \times 10''$ 88′ long.

The cross ties (sawed) are each of stuff $4'' \times 8''$ 9′ long spaced 22″ cen to cen.

The rails are taken at 60 lbs. per yard, and 28′ between joints.

The lateral bracing is not given, and I have taken the diagonals at $8'' \times 8''$ stuff, and the tie rods at 1″ diam.

Assuming both tracks of the bridge loaded, the center truss, which is of same dimensions as side truss, will carry a load equal to the aggregate load on side trusses, and if both tracks be uniformly loaded, the center truss will carry twice the load of side trusses: and with equal strength of trusses (which I understand as the case) the factor of safety in the center truss would be one-half that of either side truss.

Independent of the live load on each track, it is evident that the center truss sustains one-half the total dead load of whole bridge; or the total weight of a single track bridge of same dimensions is concentrated on this truss, hence with only one track loaded, the factor of safety of the center truss is less than of the corresponding side truss, and this of itself is objectionable, unless the factor of safety of the truss with dead and maximum live load is equal to or in excess of good practice.

It is estimated that the center truss sustains

First—Its own weight.

Second—One-half the weight of two

floor systems of equal dimensions, or the weight of one entire floor system.

Third—One-half the weight of two systems of lateral bracing, or the entire weight of one system of lateral bracing.

Fourth—One-half the live load upon either track; and if both tracks be uniformly loaded, then the center truss sustains the total load on one track, or more correctly, the equivalent of this load.

I have estimated the weight of truss alone as follows:

Lower Chord.

$$\frac{13\times12\times4\times88\times40}{2\times144}=7624.32 \text{ pds.}$$

Upper Chord.

$$\frac{12\times12\times4\times88\times40}{2\times144}=7040.00 \text{ pds.}$$

Main Braces (*sixteen*) 23.73′ *long.*

$$\frac{8\times10\times16\times23.73\times40}{144}=8436.49 \text{ pds.}$$

Counter Braces (*eight*) 23.73′ long.

$$\frac{7.5\times7.5\times8\times23.73\times40}{144}=2966.06 \text{ pds.}$$

First Pair of Rods 23′ long.

$$23\times2\times10.616\times2=976.672 \text{ pds.}$$

Second Pair of Rods 23′ long.

$$23\times2\times8.128\times2=747.776 \text{ pds.}$$

Third Pair of Rods 23′ long.

$$23\times2\times7.010\times2=644.920 \text{ pds.}$$

Center Pair of Rods 23′ long.

$$23\times2\times5.972=274.712 \text{ pds.}$$

Angle Plates, Nuts and Washers 3906.00 pds.

Total weight of truss alone 32616.95 pds.

I have estimated the weight of the floor system as follows:

Beams 17′ clear space between trusses 21′ extreme length; 35 beams sustained by cen. truss.

$$\frac{6\times14\times21\times35\times40}{144}=17150 \text{ pds.}$$

Stringers (2) each 88′ long

$$\frac{10\times10\times2\times88\times40}{144}=4888.8 \text{ pds.}$$

Cross-ties of oak (48) each track,

$$\frac{4\times8\times9\times48\times45}{144}=4320 \text{ pds.}$$

Rails, two lines, 60 pounds per yard,

$$\frac{88\times2\times60}{3}=3540 \text{ pds.}$$

Fish plates, six pair

$$6.5\times2\times2\times6=156 \text{ pds.}$$

Bolts, nuts, washers (fish plates) 25. pds.

Spikes $48\times4\times.5=96.$ pds.

Total weight of floor system 30175.8 pds.

I have estimated the weight of one system of lateral bracing as follows:

Diagonals $\frac{8\times8\times20.25\times16\times40}{144}=5760.$ pds.

Rods $2.654\times22'\times8=467.10$ pds.

Angle plates, nuts and washers 224 pds.

Total weight of lateral bracing 6451.10 pds.

The truss is of uniform (nearly) section above and below the horizontal axis, and the weight has been equally divided between top and bottom of truss.

The weight of floor system has been distributed along the lower chord, and the weight of lateral bracing along the upper chord.

The weight from truss alone along the upper and lower chords is for each panel point 2038.555 pds., and at the ends of chords 1019.227 pds. The weight from floor system along the lower chord is for each panel point 3771.975 pds., and ends of chord 1885.99 pds. The weight of lateral bracing along the upper chord is 806.39 pds. for each panel point, and 403.195 pds. for ends of chord.

The total weight of truss, floor system and lateral bracing is 69243.8 pds., of which 34621.9 pds. are sustained by each abutment.

The strains upon the rods, main braces and chords have been computed as follows:

Fourth Brace $\frac{4327.74\times23.73}{21}=4890.36$

Third Brace $\frac{12983.79\times23.73}{21}=14671.07$

Second Brace $\frac{21638.75\times23.73}{21}=24451.78$

First Brace $\frac{30294.25\times23.73}{21}=34232.50$

Horizontal strain at center of chords.

$$\frac{69243.85\times88}{168}=36270.6$$

Cen. Rods.	Third Rods.	Second Rods.	First Rods.	Abutments.
2038.555	2038.555	2038.555	2038.555	2038.555
3771.975	3771.975	3771.975	3771.975	3771.975
	2038.555	2038.555	2038.555	2038.555
5810.530	806.390	806.390	806.390	806.390
	8655.475	8655.475	8655.475	8655.475
	4327.7375	4327.7375	4327.7375	4327.7375
	2038.5550	2038.5550	2038.5550	2038.5550
	2771.9750	3771.9750	3771.9750	3771.9750
		2038.5550	2038.5550	2038.5550
	10138.2675	806.3900	806.3900	806.3900
		2038.5550	2038.5550	2038.5550
		3771.9750	3771.9750	3771.9750
		18793.7425	2038.5550	2038.5550
			806.3900	806.3900
			2038.5550	2038.5550
			3771.9750	3771.9750
			27449.2175	2038.5550
				806.3900
				1019.2775
				403.1950
				1019.2775
				1885 9875
				34621.9000

Above strains are due the dead load alone, and assuming the greatest possible load to two 36 tons locomotives, coupled on each track, or in round numbers 2000 pds., live load per foot run, then the strains on the rods and abutments become respectively,

	10138.2675	18793.7425	27449.2175	34621.90
5810.53	33000.0000	55000.0000	77000.0000	88000.00
22000.00				
	43138.2675	73793.7425	104449.2175	122621.90
27810.53				

The strains on the braces become respectively

$$4890.36+(11000\times1.13)=17320.36$$

$$14671.06+(33000\times1.13)=51961.07$$

$$24451.78+(55000\times1.13)=86601.78$$

$$34232.50+(77000\times1.13)=121242.50$$

The strains at centers of chords become

$$36270.6+\frac{(2000\times88\times88)}{168}=128461\text{-}1$$

The strains on the lower chord, considered as a beam, is for uniformly distributed load of 2000 pounds per foot, run and weight of panel, length of floor system

$$\frac{22000\times3771.98}{2}=12885.99$$

The safe load that should be put upon the several members of the truss have been calculated as follows, *a factor of safety of six* being taken in each case:

Cen. rods (*F.S. taken at* 60000 *pds.*)

$$\frac{1.29\times60000\times2}{6}=25800.$$

Third rods (*F.S. taken at* 60000 *pds.*)

$$\frac{1.50\times60000\times2}{6}=30000.$$

Second rods (*F.S. taken at* 60000 *pds.*)

$$\frac{1.767\times60000\times2}{6}=35340.$$

First rods (*F.S. taken at* 60000 *pds.*)

$$\frac{2.65\times60000\times2}{6}=47000.$$

The panel length of lower chord considered as a beam load uniformly distributed

$$\frac{13'\times24\times500\times2}{11\times6}=61459.54$$

concentrated at cen. of panel length

$$\frac{61454.54}{2}=30727.27$$

The *upper* chord considered as a pillar with squared ends, and having a length equal to one panel, calculated by C. Shaler Smith's formula for wooden pillars

$$1+\frac{\frac{5000}{(11\times12)^2}}{6^2}\times.004=1703$$

and $$\frac{1703\times288}{6}=81744$$

The upper chord is composed of *four* parallel timbers of an average width of 6″ and depth of 12″, and it is proper in my opinion to estimate the strength as that of four pillars of similar dimensions and not as a single pillar 12″×24″.

The lower chord considered as tie resisting tensile stress alone has a safe resistance of

$$\frac{312\times4800}{6}=249600.$$

The main braces by C. Shaler Smith's formula have a strength of

$$14\frac{\frac{5000}{(23.5\times12)^2}}{8^2}\times.004=837.5$$

and $$\frac{837.5\times160}{6}=22333.33$$

The counter braces by C. Shaler Smith's formula have a strength of

$$1+\frac{\frac{5000}{(23.5\times12)^2}\times.004=751.2}{7.5^2}$$

and $$\frac{7.5\times7.5\times751.2}{6}=7042.5$$

The center counter is subject to the greatest strain, and would be that of the weight on the drivers of a locomotive when on the third panel point; assuming weight on drivers 44000 pds., the strain on center counter brace becomes

$$\frac{44000\times3\times22.73}{8\times21}=186450$$

In the following table are shown the strains that should not be exceeded on

	Strength.	Dead Strain.	Live Strain.	Max. Strain.	Factor.
Lower chord in tension	249600.	36270.6	92190.5	128461.1	12.00
Lower chord as a beam	61454.5	3771.98	22000.	25771.98	14.50
Upper chord	81744.	36270.6	92190.5	128461.1	3.84
First Brace	22333.3	34232.2	870.0	121242.5	1.104
Second Brace	22333.3	24451 8	62150.	86601.8	1.56
Third Brace	22333.3	14671.1	37290.	51961.1	2.58
Fourth Brace	22333.3	4890.4	12430.	17320.4	7.80
First rods	47000.	27449.2	77000.	104449.2	2.70
Second rods	35340.	18783.7	55000.	73783.7	2.86
Third rods	30000.	10138.3	33000.	43138.3	4.20
Fourth rods	25800	5810.5	22000	27810.5	5.70
Cen. Counter	7042.5		18645.	18645.	2.28

the several members of the truss. The strains due to weight of truss, floor system, and lateral bracing, and to assumed live load.

Of course the assumed live load is greater than has ever come upon the bridge at one and the same time, but the construction and location of the structure is such that it might come upon it, and in the writer's opinion, the truss should be calculated to carry with a factor of safety of six, the weight of two 36 tons engines on both tracks at the same time.

It seems to the writer that it would be to the interest of Railway Companies, as well as to the traveling public, to employ competent engineers to design their bridges, rather than to trust the ordinary bridge carpenter who is usually charged with the design and construction of wooden railway bridges.

That many railway companies do employ first class engineers is well known, but as many more build bridges with no regard to the loads to be carried, or the factors of safety employed.

HONDURAS.—Honduras is going in for telegraphs. A line is in progress from the port of Trujell, on the east coast to the capital, a distance of 350 miles. The wire and other materials have already been sent out from the United States. When this line is completed, Honduras will be within three days of New York, *via* the Havana cable.

ENGINEERS IN INDIA.

From "The Builder."

THE engineer who has newly arrived in India has barely had time to make out the meaning of all that meets his view, or to assess his prospects, before he is placed under orders for some inland destination. The glare and heat, the peculiarities of life occasioned by a despotic form of government, the bewildering because apparently formless structure of native society, and the manners and customs of his countrymen established in the East, give a novelty which make the portals of India romantic. The new-comer is, however, at once impressed with the greatly-enhanced cost of everything. The old Indian practice of charging rupees for shillings is by no means extinct. Then the rents paid for their houses by Europeans are exorbitant. The young engineer hears of an amount equal to the whole of his salary being given for a very ordinary residence, with bare, whitewashed walls, unfurnished, and built of very coarse materials. So, with a very well-defined scale of salary, and tolerably easily guessed prospects of promotion, it is a matter of wonder when he himself can occupy one of these airy mansions at such a heavy rental. Who, again, he ponders, can be the people who purchase the wares in the shops at prices which would make his own bag of rupees, now so heavy that he can hardly carry it in the hand, disappear before ordinary wants could be supplied? How can Europeans do any work after seven a. m. and before five p. m. out of doors, when none of the Presidency folk stir during the intervening hours, except in covered carriages? Questions like these crowd on the mind. But as a matter of fact the state of British society at the Presidency towns is readily explained. It is the oldest and best-paid officials who as a rule live there, and their salaries range from £1,400 to £4,000 a year. The rate of remuneration is calculated to compensate for the expense of living, and they can afford to pay rupees for shillings. The principal Indian shops depend immensely on the up-country orders they receive. There are civil servants throughout India receiving, in considerable numbers, incomes between £1,000 and £2,000 a year; a few military staff officers are about as well off; and the expenditure by the numerous military messes gives rise to a brisk trade in domestic stores. The Presidency firms likewise act as depôts and agencies for other shops in the interior. The work to be performed at the Presidencies is, moreover, literay, and contemplative, and not of the kind calling for exertion in the sun, which accounts for the use of vehicles and the short time daily which Europeans are to be seen moving about. There is little or nothing in the appearance of the Presidency engineering offices of control to indicate the nature of the work done in the country generally. The heads of the departments, seem overwhelmed and have scarcely time for any but the briefest conversation, while there are rooms, packed with clerks, seemingly abstracted in thought, absorbed in writing, or hurrying to ask for instructions. It would, however, be perplexing to follow an engineer to the out-districts, without some idea of the main-springs which set the secretarial machinery going on its assiduous course, and the plan on which works are executed. The Indian Public Works Department took its rise in the provision of barrack accommodation for the European troops, and in the necessity for military roads to connect distant stations together. As long as the country is held by British garrisons, this work has to be attended to, and furnishes an amount of employment that is of a steady nature; for when barracks are built they have to be kept in good repair, and when roads are constructed they must be maintained. Of late years railways have superseded roads along the main lines of traffic, and though the chief railroads are not directly worked by the state, the statistics are compiled by, and the audit of the accounts rests with, the Public Works Offices. Hence a great deal of engineering routine has to be disposed of in India, as in other countries where roads and railways are among the defences of the empire. But the East India Company was a trading corporation, and the ele-

ment of speculation entered largely into their system of public works. The agriculture of the natives is carried on mainly by cottiers, working upon borrowed capital, and not by substantial farmers, so that raisng a crop with the least expenditure of labor and manure is essential to the ability of such cultivators to pay a high land-tax. In the south of India rice is a crop which meets this obligation. By turning the silt-laden river-waters on to the flat lands of the deltas, or by impounding the wash of the rains in tank reservoirs, for the saturation of low-lying patches in the valleys, a rice crop is raised with the very minimum of trouble and outlay. Land thus irrigated can afford to pay 5 rupees an acre, against the 1 rupee or 2 rupees which the people can pay who have to till and manure their acres to get a crop; in consequence there has been an increasing official effort to lay additional land under rice cultivation. No Government ever, it has been urged, had such facilities of increasing its revenue as the Indian, able to get two or three times the rental from irrigated as from ordinary cultivated land; but, like all other schemes offering unlimited profit, with comparatively small outlay, it will be found that the increase of revenue from the irrigation of rice is only obtained by the depression of the other branches of agriculture, which are better worth encouragement. So much is this felt to be the case that irrigation works are not now permitted to be termed agricultural. But a very large part of the duty of the Government engineers is connected with the state-protected growth of rice and other irrigated produce. Another peculiarity ofI ndian public works is their almost exclusive dependence upon the budget of the year. There is no certainty that a work once begun will be carried through. Provision is made year by year, and no arrangements lasting beyond twelve months can be definitely assured. This introduces a curious hesitation into the Department, gives rise to constant reorganizations in the different establishments, and a fluctuation in aims and operations that is to be seen in no other Indian department. As a matter of fact, Indian public works are a much simpler matter to consider than is usually supposed. It may startle some, but the fact is that very few large works have ever been undertaken by the Public Works Agency prior to the very recently-constructed State railways. This is easily seen by the trifling number of engineers who have received any public decoration for the works with which they have been connected. There are only two or three names that rise to recollection.

The large works of India are,—1. The Grand Trunk Road, from Calcutta to Peshawur, which, however, was opened up in sections, under several officers, and even distinct local administrations. 2. The Godavery weir and canals, as well as the weirs and canals in the Tanjore and Kistnah districts, for which Sir Arthur Cotton has received the principal credit. 3. The Ganges canal planned by Sir Proby Cantley. 4. The lighthouses on various reefs, such as at Alquada, on the coast of Burmah. The engineers who have taken part in these constructions are the persons who, either at present or till lately, have held the leading situations in the Public Works Department. It is not that a variety of useful works have not been carried out in India reflecting credit on a considerable number of engineers, but that the above are the only really large works which have been calculated to confer fame. The few extensive works enumerated were quickly superseded in importance by the guarrenteed railways, which do not belong to the class of works executed by Government agency, but quite take the wind out of the sails of the Public Works Department. They have been made a mere matter of business under London Boards, and are not thought of when Indian Public Works are the topics of discussion. The engineer, before landing, has heard so much of the requirements of India, and the large sums spent annually, that his expectations are raised to a pitch that the actual operations of the Department, as seen on arrival, are likely to disappoint. Of course it must be so when there are no works of any magnitude in course of construction by the Department of Public Works, and grants are limited by the budget of the pasing year. Before entering further into the nature of Indian public works,it is advisable to take a look at the interior of British India. There is a great sameness all over the country. The stations containing Europeans are of

two classes; those which are cantonments for troops, the rest, which contain only civil officials. Each district contains one or more of these European positions, forming usually the headquarters of the senior engineers. Military cantonments are of different sizes. The largest contains a general and his staff, a military magistrate, the regimental messes, a club of some sort, a Freemasons' lodge, some shops, and a few pensioned warrant-officers and soldiers. What distinguishes the occupation of the military and civil servants, and imparts their distinguishing bents, is that the duties of the former are exclusively confined to the limits of the station. There is a sort of tie between the military residents and the native population of the districts. Every locality beyond the station boundary ranks as the Jungle. Their associations are peculiarly English, and after the morning parade of the troops, the chief consideration is a proper style of dress, a smart social code, paying and receiving visits, and the evening promenade and dinner. Shooting parties are now rare, traveling is mostly confined to railway journeys, and leave is taken either to a hill sanitarium or to England. Conversation hardly turns upon Indian matters, as what affects the native population is not brought home in any way to the members of regiments or the staff, Indian politics run so much now-a-days on frontier questions, and the European army has nothing to do with the affairs of the immediate neighbourhood of the inland stations. The civil servants live close to the cantonment where there is one, having very often a large native town in close contiguity, which furnishes a fertile group of police cases, and is commonly a hot-bed of litigation. The senior civil officials are much tied down to headquarters, and form an integral portion of the society of the station; so that it is only the junior officials in civil employ who move much about, and see the natives in their own villages. In the smaller stations where there are no military forces, the European community consists of the district judge, the collector of revenue, the joint magistrate, the assistant collector, the chaplain, the doctor, the officer of police, and the executive engineer. In this case, moving about the country is a relief from the tedium of every-day existence, and a society entirely composed of hard-working preoccupied men and their families. Native opinion is more noticed, the contact with natives is closer, and their doings are objects of curiosity and interest. Punctilio is of less moment, and habits of simplicity are encouraged. Altogether, British influence of the soundest type is wielded from these small stations, and they are free from the scoundrels and riff-raff who collect round the bazaars where there are troops. The engineer, after joining at head-quarters, rapidly finds that the Public Works Department consists of but two classes—secretaries and overlookers. This is, perhaps, most noticeable on what are termed the ordinary public works. The State railways form a separate category, and as they are the only works of any size entered upon, with the determination to carry them to completion at a regular pace, may be considered as lying out-side the domain of public works proper. The latter consists of the roads, bridges, barracks, public edifices, harbors, drainage, navigation, and irrigation canals. There are scarcely any positions in which an engineer is exclusively engaged in designing works to be carried out by contractors. The engineer who plans is at the same time called upon to manage the financial details of construction, to hire, control, and pay the labor. There are no means of advancing an efficient engineer except by making him into a secretary. The reason of this is that there are no large works on which the services of an engineer are required for the mere preparation of the designs, and advice during construction. India is divided off into a dozen different provincial administrations, each presenting a Public Works budget. Certain sums are sanctioned annually for each province according to its revenue and extent, and then the Local Public Works secretaries decide how these grants will be spent. The provinces are divided off themselves again into civil districts, under collectors of revenue, and to each district an executive engineer, with probably one or more assistant engineers, is posted. The districts have their annual allotment out of the budget. The exact manner in which these allotments are to be utilized is a very complicated matter of arrange-

ment, giving rise to a great deal of secretary's work. But when once made, the actual expenditure is closely watched by a series of elaborate checks. The executive engineer is permitted the very smallest amount of discretion, and has to refer for constant instructions to his secretary's office. It is a *sine quâ non* that the budget grant be expended within the year, otherwise when a per-centage is struck of how the charges for salaries compare with the outlay upon works, and a heavy lapse, as it is termed, has raised this, there is a discussion as to a reduction of the number of engineers by transfer to a distance. The civil districts are subdivided in their turn, and of one of the subdivisions a newly-arrived engineer usually assumes charge. Upon coming out to India he perhaps expected to join some large work in progress, where there was a call for professional science, heavy calculations to make in office, elaborate plans to be drawn, and lines of section to be leveled and laid down on the ground. Instead of this, he takes up his quarters at some native town in the center of his subdivision, and is immersed in the expenditure of numerous petty grants upon road repairs, revenue buildings, and rural irrigation works. Possibly there is a small police-station, court, or school house to be built. To aid these are a swarm of low-paid natives of inferior caste, and one or two native clerks experienced in the voluminous accounts of the Department. The system gone upon evidently is that no one except a European is capable of being trusted with money. Consequently great stress is laid on the engineer making payments with his own hands, it being presumed that native palms are so sticky that much of the public works money would never reach the actual laborers, unless disbursed by a European or under his close check. It has to be remarked however, that the Public Works is the only department in which native agency is completely distrusted, as the collectors of revenue do all their business by judiciously distributing it to native coadjutors of various degrees. The engineer stationed in the district compiles, at stated times, his estimates for all kinds of works proposed for the year, and submits them for sanction. It is a cardinal and excellent principle of the Department that no outlay of money can take place except upon sanctioned estimates. The preparation of these documents, however, is a formidable matter in some cases, when minute details are demanded for works of a simple kind, and in every item having to be entered on paper before it can be reduced to accomplishment. The smallest deviation from programme has to be explained by letter. So that there is probably no country in the world in which the simplest piece of work gives rise to so much wielding of the pen and use of ink as India. There are several things which combine to enhance this peculiarity. It takes four natives at least to perform the same task as one European laborer or artisan, so that ledger accounts of wages have necessarily quadrupled the number of entries and bulk of English contractors' accounts. Then there is the custom of a monthly settlement of claims, which is universal throughout India. Most of an engineer's returns are monthly, so that here again clerical accumulations are bulky compared with quarterly returns. Another reason for so much writing being resorted to in India is referable to the climate. It is impossible to be out of doors for pleasure between the hours of nine a. m. and four p. m. The sun is too powerful, and produces highly unpleasant sensations in those exposed to it between those hours. It is the case, speaking generally, that throughout the year every one who possibly can stays indoors between the forenoon and afternoon hours. What, then, is to be done to fill up this space of time with anything resembling engineering work? Receiving and answering a shoal of letters exactly fills the vacuum, so that the table of every Indian official is crowded with posted envelopes every morning, according to the extent of his jurisdiction. If these did not come in he would have the obnoxious sensation of having no ready-cut out day's work to perform. Concede certain powers to assistant engineers, and certain higher powers to engineers of districts, and there would be few references to the secretary; but abridge such concessions, and a deluge of correspondence invades the secretariat, because it has to be recollected that the sums of money spent on individual works are really in themselves very moderat·, and the works now undertaken are of inconsid erable magnitude, and scattered over an

immense area. There is little a person of ordinary experience cannot decide on the spot without asking for a further opinion; but under centralised Government the chain has to be pulled through every link before effective work can be commenced.

It is rare in India for an engineer of any class to be put in charge of a single work capaple of occupying his entire energies, such as a line of road, a bridge, or a fine building. He must take up a charge of a certain extent in square miles, and do the best he can for any work that chances to fall within those limits. By great good luck there may be some sort of house in the locality he can rent from Government; otherwise the engineer has to reside in a tent, and, when moving about, lodge anywhere he can, in tool sheds or native buildings. A diary of all inspection excursions is kept, and sent monthly to the local secretariat. As Government gives a solatium in the form of traveling allowance, about ten shillings a day for short journeys, and one shilling a mile for long and rapid tours, these diaries are closely scrutinized. They al so serve to show how far payments have been made by the engineer himself, and whether the vigilance that is supposed to keep the natives from idling and peculation has been exercised. Indeed, the engineer observes that his repute as a departmental agent rises the farther and the oftener he travels. He is able to write from day to day about a fresh item of work, and this is constantly before the notice of the secretariat, either inviting positive instructions or suggesting instances of the value that his presence has been in furtherance of work and prevention of abuses.*

Traveling about an Indian district is somewhat arduous for a European. A very young man, of strong constitution, rather enjoys the shifting from place to place. He need have no elaborate furniture at his head-quarters; and, after the first expense of tents, cart, and horses has been incurred, will live very cheaply with the assistance of his traveling allowance. Indeed, it only pays if he is constantly on the move; and as, in addition, he ranks by sheer sitting in the saddle as a most energetic and rising engineer, there is every inducement to move camp as each third day comes round. But, after a time, this sort of existence becomes monotonous; and since promotion means little more than leading a similar marching life throughout a larger district of country, the engineer is apt to be struck with the strangeness of the professional groove into which he has subsided. An Indian district charge of any sort, which involves a very small and uncomfortable house, situated near a native town or village, without a European within many miles, is particularly unsuitable to a family man. Families of English residents are totally unable to stand roughing and exposure to the Indian climate. Even those of the better orders of natives have to be secluded carefully in the best shelter they can afford, and travel as seldom and as comfortably as possible. So that a district engineer, who is constantly traveling, is therefore just as constantly away from his family. He loses the profits of traveling allowance, having to keep up a proper and very expensive domestic establishment, the enjoyment of which is, however, interfered with by having everlastingly to scour the country or patrol the roads. An engineer with a family has great difficulty in rendering the monthly tale of bricks. He can only earn the distinction of being an active official by absence from home, and leaving his domestic circle to amuse themselves as they best can. In the present state of India, where large works are few and far between, this must be the position of several members of the Public Works Department. It is a desirability occurring to the same extent in no other country, and has in certain cases been so intolerable that engineers have thrown up their appointments rather than continue under the disadvantage. The Public Works Department, therefore, has, barring charge of a few military stations, and one or two appointments at the Presidencies, only the posts in the secretariats to offer to those engineers so circumstanced as to require to lead what is styled a settled life. Actual engineering work, to be remunerative and least irksome, therefore, in India, demands a monastic proclivity in the case of those engaged in its superintendence. The question at once arises whether the system of the Department is in keeping

with the age. That an engineer should run the gauntlet of a stigma of inefficiency or insouciance because of family ties is hardly capable of defence. Yet so long as native agency is entirely distrusted, and it is insisted upon that a European engineer must measure and pay for every work done out of public money in every British district of India, constant wearing, peripatetic movement will be the rule, and tranquil employment at home the exception, and the Department present an unequal field to its secular and regular orders of engineers. The difficulty is in part surmounted by the extensive employment of military agency in the Indian Public Works Department. What remains to be noticed forms the substance of a very old and still-continued argument. It is, What public works ought to be done by what is called the Civil as distinguished from the Public Works Department? To understand the merits of the controversy, it is necessary to reflect that till more than half this century had passed there were no civil engineers in the service of the Indian Government. That service was entirely composed of two classes of Europeans, divided off into members of the naval or military establishment, and those of the covenanted civil service. The rest of the officials in the subordinate employments were of the uncovenanted class, or mere natives. Civil authority, of course, fell to the share of the covenanted civilians, but the public works became a sphere appropriated to military engineers specially trained for the peculiar duties to be performed in the East. The civilian magistrate of a district exercises almost all the functions of an English property agent, from the fact of the State owning the fee-simple of the land. Hence, civil officers have been in the habit of imitating and often partially superintending those minor works which rank as estate improvements. The border line has never been very distinct, and civilians without any acquaintance with the laws of statics or the strength of materials have attempted bridges and buildings beyond their designing powers. However, a magistrate seldom directs engineering operations personally. He gives general instructions and a grant of money to the native officials under his orders, the main part of whose regular work is collection of the revenue from the peasantry and prosecution of crimes. These underlings employ a class of master-craftsmen, who have that intuitive sense of proportion so often seen displayed by comparatively uneducated men, and which is as innate and real as the faculty of music. Their constructive ability is heightened and regulated by a practical acquaintance with the execution of work handed down from father to son. All structures for the use of the natives themselves are raised by their operative instrumentality, and it is surprising what these master artisans, whose monthly pay may be only from 12 to 20 rupees, can turn out when under effective observation. The revenue officer is known to have absolute control of funds, and no higher authority to consult except in a merely formal manner as to their application. So that his orders are implicitly carried out, and all he has to do is to ascertain that a work has actually been constructed, to pass the accounts. The revenue officials have also certain powers of exacting compulsory labor on emergencies. On the whole, for short simple roads, and for buildings to be occupied by native members of the administration, and also for small irrigation works and municipal improvements, execution through native foremen directed by the magisterial collectors of revenue, is a cheap method of construction, and amenable to scant objection. The advocates of every kind of district work being placed under a European engineer of the Public Works Department assert that, although the works may be simple, there is no check against peculation, and that the style of work and its cost are left entirely in native hands, with an uncertainty as to its quality. There is little doubt that the natives, when given discretion, do not pay labor with the same regularity as a European, and that small sums are appropriated and relatives provided with petty positions on the works. But if any one will be at the pains to reckon up the probable amount of such *laches* in a year, and place them against the cost of the establishments which would have prevented their occurrence, the difference would probably not be in favor of the change. Where civil revenue officers

fail is in attempting rather heavy works with inadequate means. Flimsy edifices and threadbare bridges result, which show the want of professional training. The natives do not venture to dispute a positive order, and thus works are run up without check, in the enthusiasm of an otherwise successful tenure of office, which bring the brief approbation of the distant Board of Revenue, and disparaging comments some years afterwards, when the tents of the Provincial Governor and his secretaries on tour are next pitched in the vicinity. The remedy, of course, is that district civil officials should submit all designs for the scrutiny of a civil engineer or architect possessing local acquaintance, leaving the method of actual construction a question of economy and convenience. The Public Works Department has a totally different way of executing work to that pursued by civil district officials. Its whole organization is on a European model, and native routine is looked at as slipshod and irregular. The engineers have a fixed staff of college-educated subordinates, very much on the plan of the French *Ponts et Chaussèes*. Natives, as a rule occupy the lowest positions, and there is nothing that a native does which is not presumed to be checked by a European, either in the field or office. The Europeans are themselves overawed by a network of returns and a watchful secretariat. It is well known that a native in the ordinary civil departments is not satisfied with the salary of his appointment, but makes it worth much more by indirect acquisitions. The mode of doing so is kept out of sight, and is often merely the exercise of influence and patronage, which was the reward of political eminence only lately in Great Britain. At any rate, ample opportunity exists in all Indian situations held by natives in the offices of legal and revenue administration for making more than their nominal stipends. So much depends upon custom, and is readily conceded by the population, that the practice can neither be inquired into, nor stopped. For example, it is quite possible for a subordinate deputy to draw a traveling allowance and yet quarter himself upon the people wherever he proceeds, and be put to no expense. But in return for this entertainment the landed classes expect to secure a friendly ear at the Revenue Office, and that their disabilites will be examined with extra care. So again the native contractors, and master tradesmen, who are entrusted with district works intend to supplement their periodical remuneration by an occasional windfall. But the natives in the Public Works Department, though tolerably well paid, are under such a strict watch, and render such telltale accounts, that they can make nothing without the fact being either guessed or known. Consequently they have to be driven to exertion, and take little or no real pride because they derive so little satisfaction from their work. They can nominate no relatives to petty charges, nor can they assume among their countrymen the importance of sole managers of public undertakings. The effect is that every arrangement has a tendency to go wrong through neglect, where the European engineer cannot be constantly present or frequently around. There are many ways in which a native member of a permanent establishment can have his own way with impunity. One of the easiest is to feign stupidity. It is exceedingly difficult to displace on such a ground men who have been trained at a Government Institution. Then there is sickness, and the ability to account in the most plausible way for a scarcity of hands and dawdling progress. But very often the native subordinates, though well educated, are really defective in practical art, and quite unable to direct workmen. So that an engineer has frequently several sub-agents whom he is obliged to employ, but who may be neither competent nor attentive. It then comes to this; that if the work is a large bridge or a single line of road of moderate length, or a building which the European engineer can visit every other day, the departmental system is much more regular and efficacious than the management of the collector. But if, on the contrary, the works are of a petty description and very widely scattered, the labor of check and the physical injury to the European constitution, caused by incessant exposure to a tropical sun, are dearly purchasd by any ecomony or benefit produced. That this is the case is not readily seen in the secretariats, by whose opinion Government is chiefly guided. A secretary directing public works from a desk has to secure that some person in each of the districts

is responsible for whatever goes amiss. Once an engineer is placed in charge on the spot, dealing with him becomes a mechanical matter, and if anything in the shape of loss, failure, or short expenditure occurs, which he is unable very fully to explain, he is liable to be handed up to higher authority. Now a capable engineer employed upon a large work which he can go over from end to end without great fatigue and exposure, or long absence from his ordinary residence, or upon which furnished houses have been built every ten or twelve miles by the foresight of Government, can very seldom fail in explaining casual accidents or unlooked-for difficulties. But when he is made responsible for every little rut in a road and leak in a sluice over a wide district, which it would take three months to go round at the pace of ten miles a day, it is very different. When, again small losses and instances of disrepair are treated as professional delinquencies, it becomes exceedingly difficult for an engineer to ward off censure. Some civil official has perhaps drawn an unfavorable comparison between a collector's and departmental work on return to head-quarters from a district tour, the smart of which is felt at the secretariat. The engineer has to be asked pointedly when he was last over that corduroy mile of road. If by a happy accident a recent visit had been paid, some means of pacification are promptly forthcoming. In the other event it would need much ingenuity to get the blame saddled on the right horse. So that the position of a district engineer having a number of insignificant works to look after is one of actually much greater anxiety, far more wear and tear on his physical endurance, and discomfort, than if engaged on a single important work involving four or five times the expenditure of money. The only safe procedure is to be constantly on the move. The impossibility of being in more than one place at a time can thus always be urged, and is an unanswerable plea, and a sufficient exculpation. The district engineer who is scarcely three or four days in the same neighborhood, can look with tolerable complacency at occurrences which are not at all relished by another, whose less frequent journeys do not furnish the grateful *alibi*. Members of the Public Works Department will be found to assert the superiority of the principle of having every work in the limits of a district in charge of a European. There are persons, again, who consider the mere fact of an educated gentleman traveling about and mixing with the natives an immense factor in the propagation of civilization, and they would advocate the engineer moving about even if he had a mere trifle to inspect. This is curiously evidenced by the different treatment two individuals would receive in the disposal of their leisure. A sportsman might take two or three days shooting, and be thought all the better of for thus occupying his time. There would be no inquiry as to the interlude in more serious labors. But a reading man who for purposes of study had halted for the same period, would experience little appreciative encouragement. He would probably be set down as wasting time. The idea evidently is that the huntsman is coming in contact with the natives and improving them, whilst the other is merely invigorating his own mind and producing no impression on the native population. But this is a mere relic of the company's system, who governed the country on commercial principles, and of a period before railways, when comparatively few had a knowledge of the interior. At present the main reason for extending the supervision of a European over small district works is, that by this means alone can the expenditure on those works most commonly done largely exceed the cost of salaries. District works have so little finish, and are designed upon such an economical scale, that it takes a considerable number to fill up an annual budget respectably. But it may be questioned if there is any absolute reason for an engineer having to undertake every class of work within a certain specific area before he becomes entitled to receive sufficient remuneration for his services, any more than that a judge should be paid so much per judicial sentence. There must at times be a lull in engineering enterprise, while an Indian district always presents some important schemes to investigate and a good deal of scope for land improvement. But to curtail an engineer's breathing-room by exercising pressure through the diary of movements and occupying his time in tedious journeyings, in order to visit works which the natives,

when the point is insisted upon, are perfectly competent to manage unaided, seems to approach a misapplication of a European's peculiar powers and qualities. District charges are much more onerous in some parts of India than in others. In Bengal, the North-West provinces, and the Punjab, only the principal lines of road are metalled, the canals are in the hands of a distinct group of engineers, and on both classes of works there are small two or three roomed houses at intervals of about a dozen miles apart, which shelter the traveling engineer from the mid-day heat. The population is often as dense as 400 per square mile, and the districts are not extensive. Twenty or thirty miles take a person from the head-quarters of one district to the principal station in the next adjoining; also in the Bombay presidency the district divisions are not excessively large, and the works consist chiefly of trunk lines of road, or buildings in the forts, towns, and military stations, or marine construction. It is as the south is reached that the change in the system of collecting the land revenue makes itself felt in a minute interference with almost every acre of the country on the part of the Public Works as well as other agencies of Government. The population dwindles down to 200 per square mile, and the traveller may go 100 miles without coming across a residential community of Europeans. Except on the military roads there is no accommodation provided for an inspecting engineer, the Madras Government, even on the important irrigation canals, grudging the small expense that would be required to erect the houses which are the foremost items of construction in a work of any magnitude in the North of India. Obviously, to leave Europeans, as a rule, and from year's end to year's end, to locate themselves during the hot and rainy seasons in tents which are, except for very temporary occasions, inadequate shelter, or in flimsy mud-huts they subscribe for, little bigger than tool-sheds, is uneconomical. It prevents them using their best faculties, feeling they have a full command of the country, or preserving undiminished health. Few professional men in Great Britain can imagine what it is to be encamped in the hot weather, directing a public work from a tent. The mornings and nights are endurable enough; but the hot, dry winds and dust of the middle of the day evoke such uncomfortable sensations that enforced repose under blankets is the most suitable manner of spending the greater portion of the hours of mid-day. The high temperature, coupled with the electric tension of the outdoor air of an Indian midsummer, cause such an oppression as almost to stifle thought and render any clear exercise of the head and hands of the engineer a labored and forced performance. In a shaded and moistened apartment, however, the conditions are materially altered, and an individual can do a fair amount of work, compared to that of the brief waking periods under canvas. During the cold season, which lasts for half the year in Northern India, tents are very agreeable, and their use is indispensible on cross-country trips of any length, and when breaking fresh ground. But they are cumbrous to move about, and ill-suited to hilly or roadless districts. In the Madras Presidency, where it is nearly always hot weather, the mere apology for a cool season only lasting for two months, it is hard work merely inhabiting tents, to say nothing of preparing designs and getting up intricate engineering schemes under the sweltering awning. It comes to this, that when works of novel construction, or of considerable size, have to be undertaken in India, it is desirable to employ a European staff and to protect them from the effects of climate at any reasonable cost. But if the works are small and widely diffused, and such as the natives are able to execute with far greater cheapness and speed than any European, if they only choose, it is of questionable utility to compel Europeans to direct the operations with an expenditure of motor-power, that only the youthful can long stand and the restless enjoy. There has never been much attention paid in India to distributing engineering work in proportion to the competency, time, and powers of each member of the department. The first effort has been to perfect a chain of rules and form an inflexible system. This done, item after item of work has been brought within the vortex by simply including it in some one's charge who was bound to observe the departmental regulations, till the whole tone of the Indian works became one of over-tasking.

Nobody could do all that was expected of them, so that it has been a matter of forbearance or accident when any lengthy period has passed without the engineer actually executing the smaller class of works and repairs getting a reprimand. Till the close of the American War of Secession, public works were being very briskly pushed on in India, under the surviving principles of the East India Company. The country was just being opened out, and the comfortable lethargy of palanquins invaded by the road and the railway. It was expected that the natives would rapidly adopt European ways of living, and exchange clay and thatch for stone and brick. Irrigation was to spread fertility, and pay investors 10 per cent. Sanitation was to dispel miasma. So that in every quarter of British territory the engineer was about, and the most stirring personage in his neighborhood. No official worked longer hours, or went into such unvisited corners of the district. At last the more obviously needed works have been completed, and those only have still to be carried out which form the demand of a very advanced country. Yet there is little encouragement to proceed with improvements, because the wealth of the population and the productiveness of the land have not kept pace with the extended facilities of locomotion. They have opened a market, but the first rush of raw produce has been succeeded by a dearth, which has caused the failure or decline of many of the best mercantile interests at the presidency towns and inland settlements. At present there is no steady flow of capital to engineering works. Grants are obtained with considerable difficulty, and it is only where some political emergency favors a scheme that it is capable of furnishing high-class employment. The whole structure of native society operates against rapid material expansion. This is seen at a glance by comparing the native town with the European station close beside it. The one is a compact mass of badly-built houses, mostly single storied and in which there is more clay than any other substance used in construction. Each room contains its four or five human beings, and the whole city may have from one to two hundred thousand inhabitants. Every article in the possession of the natives has a rickety appearance, and the problem uppermost is how to live on the smallest possible sum. If people are seen talking in the streets it is almost a certainty that they are discussing the price of food. Houses of the natives are generally built round a small central court-yard. They seldom contain any furniture, and the walls are about 8 feet high. The several generations of a native family club together in the same establishment, so that there is no individual independence. The prosperity of one in the household means that the others will do so much less work. When a man obtains a Goverment appointment all his relatives flock to quarter themselves upon his bounty. Early marriage is the almost absolute rule, and the natives, as a body are not long-lived. The inferior cheap garbage on which they live, the filth of their towns, and the weak constitutions and indolent habits most of them have inherited, all conspire to shorten existence. A native is extremely old at fifty years of age, and a woman at thirty five. The rate of mortality in places like Calcutta is 59 per 1,000. In the Sepoy Army, where selected men are under European care, the rate is as low as 14 per 1,000. These several facts produce a great influence on a native's way of thinking and acting. They are essentially different from those of a European. The British station is an enormous contrast to the native town. Neat plastered houses, constructed to admit a flow of the purest air, and of very large size compared to a native house, which last is on the scale of one of its stables or store-rooms, stand in an ample enclosure. There are from four to eight commodious rooms in an English house, and the whole are appropriated to the use of but two adults. The accomodation for 100 European residents covers as much ground as a native city of fifty thousand people. So far from being contented with a subsistence, the European is striving to obtain a competence. He affects some elegance in his furniture, tidiness in his grounds, and cleanliness reigns in the neighborhood of the bungalow. The roads in the European settlements are well kept, and it is, in fact, a complete contrast to the squalid purlieus which the natives not only inhabit, but really like. Whether all do or not, their circumstances prevent any important assimilation to European habits,

as it was at one time expected would occur through education. Natives cannot be Englishmen. Their brightest time is in the immaturity of youth. There is no dignified evening of their days to be provided with substantial comfort and intellectual ease. The ambition of a native is to escape all kinds of labor that involve physical exertion and exposure to weather, by obtaining a public appointment, after leaving school, on a small monthly salary, with a prospective and still smaller pension. Natives, however, while their faculties last, are both clever and industrious. The only thing is, that they must be allowed to produce results in a different way from Europeans. They distribute a given task among a number instead of placing it in the hands of a sole person. This is of great moment to notice from an engineering point of view. Hitherto natives carefully educated as engineers, and who have passed high mathematical examinations, have been treated on entering the service as if they were Europeans. In consequence they appear much as fishes out of water. It is impossible to expect the same physical endurance in a native engineer as in one brought out from Great Britain. Nor will a native who will wear out at fifty, have the same kind of ambition spurring him to effort as the Englishman, who is still in full vigor at that not advanced age. Besides this, the native is not a free agent. He is merely the member of a large house-and whose slave he virtually continues hold whose wishes he is obliged to consult, while in employment. So that he is continually, work how he may, being reduced to mere subsistence level, and has no scope to found a career and domestic circle purely his own. If, therefore, a native is departmentally employed in the higher ranks of engineering, it is necessary to allot a diminished charge, and useless to pay a scale of salary which ascends by rapid leaps.

But under a purely native system of government, which would keep an engineer in one locality the whole of his service, while, on the contrary, British principles of administration would remove him to a fresh district every three or four years, an engineer would so establish himself as to be capable of controlling the same, if not more, district work than a European. The native would collect his relatives together, have all the most likely males well schooled and taught the elements of the profession, and then make use of them as assistants to himself. The head of the family would alone draw pay from the State as engineer; but his duties would be shared by several relations qualified to help, but whose names would not necessarily figure on the establishment-rolls. Other dependents of the family, possessing no education, would, notwithstanding this disadvantage, be turned to account. They would be posted to the different works as spies on the doings of the subordinate agents. In this way the native engineer would be able to exercise a very powerful check on irregularities of all kinds, and be prepared to make efforts with which it would be most difficult for a European to compete. The Public Works Department is so constituted as to prevent the formation of this sort of local influence. For want of it most of the native engineers appear helpless, and are liable to be set down as failures. But the civil business of the country is done by these convenient socialisms, though ostensibly only one responsible official is dealt with,—the magistrate-collector. It is here that the Revenue Department is stronger than that of Public Works. Agents intimately acquainted with a district, and long settled in it, can get work of an ordinary description executed with more certainty and less cost than Europeans, who do not live among the natives, and cannot hear what transpires. But these observations only apply to scattered local works, and not to costly single projects requiring science and organization. The natives do not posess much administrative power. They make haphazard arrangements and leave chance to do the rest. In these cases to have engineering work efficiently carried on, Europeans must be at the head of affairs. Natives have not had the opportunity of acquiring the necessary experience, and they have neither the perseverance nor comprehensiveness of grasp to control extensive undertakings and meet unexpected difficulties. It is just for the reasons that make English Engineers in request all the world over that some of them are needed in India. But it has to be recollected that it is not an opulent country, and will not for long have very many large works to offer.

ON THE APPLICATION OF THE METRIC SYSTEM TO THE STEAM ENGINE.*

By ALFRED COLIN, M. E.

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MEASURE OF POWER.

The power of steam engines is estimated in horses; the expression chosen by Watt a century ago, as a commercial but not as dynamical unit was adapted to his purposes. Since then the Steam engine has been called upon to perform works which at that early period were not thought of, and the expression Horse-power has rapidly lost its original meaning. Considering that industry will constantly require increasing powers it is proper to see if some more appropriate unit is not desirable, and the introduction of the Metric System offers an opportunity to study the question.

A horse-power is defined 33,000 foot-pounds in a minute, which are equivalent to 75.73 kilogrammeters in a second. In the Metric System the horse-power is reckoned for the sake of simplicity at 75 kilogrammeters. Consequently when the power of an engine is to be determined, its work is calculated and $\frac{1}{75}$ of this quantity is the horse-power, in other words the coefficient $\frac{1}{75}$ is introduced. A slight consideration shows that this is a very indirect process, inasmuch as the real unit is the kilogrammeter.

The work of an engine may be measured in two ways; that performed by steam on the piston or the *Indicated Horse-Power*, of easy calculation, and that transmitted by the shaft which is the *Effective* one. This last in early engines was computed with a Prony's friction dynamometer. A considerable discrepancy exists between these two expressions of the power, and no relation can be formulated; but from this discrepancy arise the complicated formulæ with which treatises on the Steam engine abound. It would not only be useless but impossible to try to enumerate these manifold empirical formulæ; and, although this constitutes a subject of complaint from every engineer, complications still arise every day by the general introduction of the compound engine.

In Europe this is due to the term "nominal horse-power," according to which engines are sold. In this country we employ the only criterion which presents no ambiguity—the dimensions of the cylinder, diameter and stroke, and yet, by an inexplicable reason, we measure the one in inches and the other generally in feet.

If now we inquire how many people have an idea of the horse-power of an engine or boiler we shall find that these are surprisingly few. Stationary engines are sold for engines of a given power, always with as high a figure as possible, and most often neither buyer nor seller is able to estimate this power; this is particularly the case with boilers. If due attention is given to this consideration it will easily be seen that the change of unit will afford little inconvenience, inasmuch as those who are conversant with the matter will find in it a source of simplification.

Years ago Mr. Reech, the Director of the School of French Naval Engineers proposed to relate the power of engines to the decimal system. He was urging then to make the horse-power 100 kilogrammeters, which would give about 75 on the shaft, or still better to take a new unit of 1000 kilogrammeters. This new unit which presents many advantages I shall call a Tonmeter*. It is very large $(\frac{1000}{75}=)13\frac{1}{3}$ actual horses and on this account may be objected to. However when we consider the great power developed either by a locomotive, a marine or a pumping engine and find its advantages, we can indeed readily infer the ratio of the tons of freight, of the tonnage of the ship, of the cubic meters of water pumped out to the ton of power. In fact in whatever way we may consider the working of an engine, coal burned, water evaporated, the tonmeter is always simpler.

As to the proposed term *tonmeter* it is in accordance with the metric table of weights, needs no explanation to be understood, and the coining of some new tech-

* Paper read at the spring meeting of the American Metrological Society (May 18, 1878).

* The French word would be *tonnemètre* and the Spanish *tonelametro*.

nical term taken from the Greek does not appear expedient.

In the study of thermodynamics, where large numbers are dealt with, the term tonmeter is particularly acceptable.

The only objection to the expression tonmeter is when small powers are required, but then a fraction such as $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{1}{8}$ or rather a decimal of the tonmeter is available, and besides the kilogrammeter the real unit, may be resorted to.

If consequently a new unit, now as when Mr. Reech proposed it, presents only advantages it is pertinent to inquire why it was not adopted. The reason is that Mr. Reech was then ahead of his time. Europe was at that epoch greatly dependent on England for marine engines, and the expediency of having a commercial unit much at variance with that of the largest producers was doubtful. But if no steps have since then been taken to introduce a sound unit, this must be only attributed to a spirit of routine.

MEASURES OF PRESSURE.

The steam and vacuum gauges next command our attention. The atmospheric pressure in French works is estimated at 1.$^{\text{kilg}}$033grammes with the barometer standing at 76 centimeters (its average height in Paris), and engineers neglecting the 33 grammes say with an inappreciably small error that the atmospheric pressure is 1 kilogramme on the square centimeter, making thus an easily remembered figure. But French steam gauges have a scale which is not advisable, the pressures being given in *atmospheres* of 76 centimeters of mercury, and in tenths; now since nothing is more variable than the atmospheric pressure, a more arbitrary unit could hardly have been selected; besides the reduction to the pressure on the square centimeter requires a small calculation in contradistinction with the aim of the Metric System. For instance: a steam gauge indicating 5.4 atmospheres represents a pressure of $(5.4\times1.033=)$ 5$^{\text{kilg}}$578grammes on the square centimeter. When small pressures are dealt with those are given in centimeters of mercury; thus 152 centimeters indicate a pressure of

$$\left(\frac{152\times1.033}{76}=\right)2 \text{ kilg. } 066\text{grammes.}$$

The term *atmosphere* had its origin in early engines when it was from the atmospheric pressure that their chief power was derived; to-day with our constantly increasing pressures this term becomes more and more out of place.

I should advise to have the pressures indicated in kilogrammes the figures 0, 1, 2, 3 . . . on the dial standing for kilogrammes, and the spaces between them divided decimally so that 5.4 would be read 5 kilogrammes 4 tenths, but not 4 hectogrammes or 400 grammes.

I must say that of late years some engineers gave attention to the above way of measuring pressures, and recommend a change in the sense here advocated.

The vacuum guage presents a difficulty; it is always in centimeters of mercury that vacuum is estimated. Many of the engine attendants are ignorant of the difference existing between a steam and a vacuum gauge, and it is not unfrequent to hear them speak of a vacuum of 25 pounds. Vacuum gauges are graded in centimeters of mercury, and I am opposed to this graduation by the reason that aneroid gauges are daily more employed, replacing the mercurial ones, and besides the idea of pressure is the only important one. It would be advantageous to grade them as steam guages, and to mark 1 *kilogramme* the point corresponding to 28.966 inches or 735 millimeters of mercury exactly ($\frac{1000}{1359\,6}$=735.51) to divide this space into 100 parts, and to continue some 10 divisions beyond the 1 kilogramme (1.$^{\text{kilg}}$1=80 centimeters of mercury). Consequently a vacuum of 76 centimeters or 1$^{\text{kilg}}$033 grammes corresponds on this scale to 1 kilg. and $\frac{33}{1000}$, or between 3 and 4 hundredths. The divisions on this scale would be smaller, henceforth, if anything, better than the actual in centimeter being about $\frac{3}{4}$ of their value.

It is important to remark here that when indicator diagrams are taken, the pressures and nothing else are read, it being the useful thing sought.

The desideratum, it seems to me, is to bring steam and vacuum gauges to one single unit, that of weight.

In Physics, the term *atmosphere* is a favorite, for its great advantage consists in discarding the mention of a surface unit. A pressure of 2 or 3 atmospheres evidently mean that this pressure

is applied all over the surface, whatever it may be, whereas a pressure of 2 or 3 kilogrammes requires that the surface acted upon be either mentioned or familiar enough to be understood, and the conventional one is the square centimeter.

But it must be conceded that the second mode of stating pressures is clearer to the majority of minds. It applies well also to very high pressures; if water passing from the spheroidal state to that of steam be estimated to develop a force of 1000 atmospheres we shall more nearly conceive an effort of 1000 kilogrammes, or rather 1 ton per square centimeter. It is indeed greatly to be regretted that physicists should have selected such an unsatisfactory unit as the atmosphere.

THE AQUEDUCT OF SEGOVIA.

From "Iron."

The great marvel of Segovia, the great achievement associated from time immemorial with the city and blazoned in its municipal arms as its proud cognizance, is the Aqueduct, or "Bridge," as it is called—a long double line of arches thrown across the ravines of the valley of the Eresma, and forming, as it were, a triumphal arch and gate of the city, as the traveller drives under it at the end of his journey from La Granja. It is called a Roman building, and attributed to Trajan, the Emperor whom the Spaniards claim as their countryman, but it bears no inscription, and apparently never bore any; nor does any record or hint occur in ancient writers either of the edifice itself or of Segovia that can furnish a clue to the date of the building or the name of the builder. To doubt that the aqueduct is a Roman work would be little less than heresy; yet there are some native critics who timidly, and, as it were, with bated breath, venture to suggest that its original design and construction, at least, may be anterior to the Roman domination of the interior of Spain, and that it may be claimed as the achievement of those Celtiberian or other indigenous races who, like the Etruscans, Ligurians, and other native Italian tribes, knew something about architecture before the Romans, and gave their masters some useful hints in that art in which they became so eminently proficient. This opinion is grounded on the fact that the Segovian Aqueduct is constructed of large blocks of stone laid one upon another, Cyclopean fashion, without cement or mortar, in the style of which specimens remain in the walls of Tarragona, the huge stones in many instances underlying the layers of Imperial Roman masonry. Another argument urged in support of this theory is, that this aqueduct, although solid enough to have withstood the wear and tear of at least fifteen centuries, is by no means a "massive building," as the guide-books call it; it has nothing of the massiveness, loftiness, and grandeur of the arches of such aqueducts as strew with their ruins the Campagna of Rome or the plain of Merida, but it is a light, airy, fairy structure, standing on very slender pillars, almost miraculously; and the wonder and beauty consist precisely in this, that the architect, whoever he was, contrived to give his work just the strength he needed to answer the purpose for which it was intended, grudging even one pound's weight of material which might give it an appearance of heaviness and clumsiness, and clash with the general look of the monument. So striking is this masterly adaptation, this perfect adequacy of the means to the end, so flimsy, fragile, and gossamer-like are the lines of this marvelous arcade, that popular tradition assigns it to a supernatural origin, the legend being that it was constructed in one night by the devil, enamoured of a Segovian damsel, whom he wished to save the trouble of carrying her pitcher up and down the stream to fetch water. The truth of the matter is that we are very

much in the dark as to the development primitive art may have attained in ante-Roman times, among races flourishing under the blissful influence of these Southern climates. They have in Segovia a so-called statue of Hercules in a convent bearing the appellation of *Casa de Hercules*, carved in all probability at a remoter epoch than any that can be assigned to the Aqueduct; and they have also two of these *Toros de Guisando*, of which Avila boasts four, of which there were sixty-three in the Peninsula in 1598, a number reduced to thirty-seven in 1820, and which have been a puzzle to the most learned antiquaries—carved masses of granite in the shape of quadrupeds, but worn so smooth and out of all shape by time, and by the inclemencies of the thousands of winters to which they have been exposed, that it has become actually impossible to ascribe them to any living species, extinct or living; and, though they are called "bulls," they are variously described as huge pigs, elephants, or hippopotamusses, or whatever else fancy names them. These specimens of rude sculpture—probably idols of a primitive race—are undoubtedly anterior to Roman times.

I could not hope to convey any distinct idea to the reader of the magnificence of the Segovia Aqueduct, by the statement, borrowed from a very able monograph on the subject by Don Andrès Gomez de Somorrostro, to the effect that the water of the Fuenfria, running through the Rio Frio, was brought to it from the Guadarrama Mountains, over a distance of 10 or 11 miles, and that the aqueduct was made to go through several bends and turnings, to check the impetuosity of the stream, running 216 feet to the first angle, 462 feet to the second, and 937 feet to the third, where it becomes a bridge spanning the valley from bank to bank, and resting at the end on the solid rock on which stands what is left of the battlemented walls of the town. The total length of the aqueduct is thus 1615 feet, and consists of 320 arches, which begin single and low, but rise gradually as the ground sinks, to maintain the level, and become double, one tier over another, as they vault over the gap of the valley, over the stream and the highway, all along the range that faces the traveler, as he approaches to, and passes under it, entering the town. The three central arches are the loftiest, and rise at a height of 102 feet. These are on the nether tier surmounted by three layers of stone somewhat in the shape of a step, intended as a cornice to mark the locality of the town-gate, and over the step in one of the pillars of the upper tier are scooped two niches, with a statue of the Virgin in the niche looking to the town, and in the other at the back a nondescript figure, that priests call St. Sebastian, but in which the Segovians fancy they behold the effigy of the Satanic architect of the bridge. No words and no picture could convey the impression wrought upon the traveler by the sight of this magic building. The whole structure is of granite, light grey, as found in the quarry, but turned by age to a light pearl and purple tint, glowing like jasper in the deep blue of this semi-Alpine Castilian sky. The blocks of stone on a near inspection seem to have been laid upon one another clumsily and, as it were, at hap-hazard, some of them so daringly jetting out and hanging over as to suggest the apprehension that the whole fabric may at any time collapse and slip down to the ground like a castle of cards. Yet the bridge has been standing perhaps, 2000 years, and looks intact; and the design, seen at a proper distance, is a model of ease and elegance, relying, one would say, on mere symmetry and balance for solidity. The stones rudely cut in large, long, square blocks, bear the holes of the iron clamps by which they were hoisted up to their places; they are worn smooth and almost round by time and storms, but sound at the core; and at the base of the pillars, as well as at various stages up the shafts and at the turning of the arches, there are cornices of what seem to have been black marble, but now everywhere chipped and cracked and almost altogether fretted away.

The Aqueduct is the only thing really living in poor dead Segovia; the necessity of securing a constant supply of better water than what flows between the ravines of the Eresma compelled the construction of this work when the place was a mighty city, and insured its preservation as the town sank year by year to its present forlorn and dilapidated condition. The

Moors, who sacked the town in 1071, pulled down thirty-five of the minor arches but the watercourse continued uninterrupted, the people contriving by woodwork to prop up the wooden trough or pipe running at the top. The dismantled arches were restored as much as was practicable in the original style in 1483 by Queen Isabella the Catholic, and the Aqueduct has suffered no outrage from that time. What is more properly called the bridge—*i.e.*, the double range of arches across the valley—escaped even the ravages of the Arab invasion. There is something exalting and flattering to human pride in the contemplation of this edifice, which, like the Pantheon at Rome is between 15 and 20 centuries old, and yet not a ruin. One dreams of the works of men that have risen and fallen in the adjoining city, while the aqueduct has been standing and performing the humble, yet vital, service for which it was intended, unmoved by the joys or woes of the population to whose most pressing wants it ministered. The ancient cathedral begun, it is said, in the 6th century, finished in the 12th, where councils were held and kings were crowned, used as a fortress against ruthless enemies, and so ravaged that its Holy of Holies had to be removed to the spot where the present edifice rose in 1525; the Alcazar, till 1866 a masterpiece of Royal magnificence, now a mere wreck, destined to moulder on the ground to the end of time; the scores of churches, convents, sanctuaries, chapels, and hermitages, crowding the streets of the town and its suburbs, some of the Lord's houses now closed from want of worshippers, some of the fat fraternities dwindling in numbers and thinning in flesh as, if not in faith, at least the lavish charity of their patrons cooled and fainted; the fortress-palaces where proud nobles learnt valor and courtesy, then sulked and idled, and laid aside the energies and spirits which made them the bulwark and the scourge of their meaner countrymen—all that made Segovia in the middle ages and unmade it in modern times—had its rise and fall, its life and death, during the long period since the Aqueduct first threw the shadow of its aerial arches on the skirts of the rocks on which the city stands. Mighty Castilian kings with their host of iron-clad warriors; the sweet and pious and thrifty Isabella on her palfrey, with her crafty Aragonese husband and her Cardinal-Minister by her side; the popular heroes the rebels Padilla and Juan Bravo; the sallow and gloomy Phillip II., and, perhaps, Columbus and Cortez, and a host of minor notabilities, rode centuries after centuries under these arches. The records of great events—the long war against the Moors, the formation of the Spanish Monarchy, the extinction of the people's liberties, the expulsion of the Jews, the revolt of the *Comineros*, the decline of the national character, the repeated French invasions, and the re-vindication of the country's soil from the hated foreigner—all seem engraved on those slowly yet eternally crumbling stones, piled up by the constuctive genius of a man who will be for ever nameless. Those swarms of swallows and martens which hover on the wing in clouds about the lovely fabric, the shrieks of which are almost the only sound enlivening the air in the stillness of the summer sunset, have been there—they or their progenitors—ever since those arches were first bowed. The clay of their nests, hardened by ages, has clung to those stones since they were laid: generation after generation of these erratic birds have come back year by year from remote regions to their favorite haunts in those stones, and have found there undisturbed home season after season. Special correspondence of *The Times*.

The Master of the Company of Turners, according to their custom, propose to give this year their silver medal, the freedom of the Company, and, subject to the consent of the Court of Aldermen, will also obtain the freedom of the City of London for any workman, whether master, journeyman, or apprentice in the trade in England, who may send in the best specimen of hand turning in either of the following materials: — Wood, stone, iron, steel, brass, gun metal and phosphor bronze. Bronze medals and money prizes are also to be given. Full particulars may be obtained by application to the Secretary of the Company of Turners, or to the Phosphor Bronze Company, 139 Cannon Street.

—*The Engineer.*

ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION. PART II.

By THOMAS CRAIG, Ph.D., Fellow in Physics in the Johns Hopkins University, Baltimore, Md.

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III.

§ 3.

MOTION OF A HOMOGENEOUS SOLID BODY IN A FLUID.

The investigation can be divided into the solution of two distinct Problems: first; given the motion of the solid body to determine the motion of the fluid; second; to determine the motion when given forces act on the body and the fluid.

Assume the axes of x, y, z, as fixed to the body, the axes of ξ, η, ζ, as fixed in the fluid, the latter being supposed to extend to infinity in all directions, and nowhere divided into regions by bounding surfaces, giving us then a single valued velocity potential φ. Let the direction cosines of the moving axes be $\alpha, \beta_1 \ldots \gamma_3$, also the co-ordinates of the moving origin be α, β, γ, then we have

$$\begin{aligned}\xi &= \alpha + \alpha_1 x + \alpha_2 y + \alpha_3 z\\ \eta &= \beta + \beta_1 x + \beta_2 y + \beta_3 z\\ \zeta &= \gamma + \gamma_1 x + \gamma_2 y + \gamma_3 z\end{aligned}$$

The quantities $\alpha \ldots \gamma_3$ are of course functions of the time, and our velocity potential φ is a function of ξ, η, ζ. The expressions for $\Delta^2\varphi$ and

$$\left(\frac{d\varphi}{d\xi}\right)^2 + \left(\frac{d\varphi}{d\eta}\right)^2 + \left(\frac{d\varphi}{d\zeta}\right)^2$$

are unchanged if we write for the quantities ξ, η, ζ, the quantities x, y, z. For since

$$\begin{aligned}x &= \alpha_1\xi + \beta_1\eta + \gamma_1\zeta - (\alpha_1\alpha + \beta_1\beta + \gamma_1\gamma)\\ y &= \alpha_2\xi + \beta_2\eta + \gamma_2\zeta - (\alpha_2\alpha + \beta_2\beta + \gamma_2\gamma)\\ z &= \alpha_3\xi + \beta_3\eta + \gamma_3\zeta - (\alpha_3\alpha + \beta_3\beta + \gamma_3\gamma)\end{aligned}$$

we have

$$\begin{aligned}\frac{d\varphi}{d\xi} &= \frac{d\varphi}{dx}\frac{dx}{d\xi} + \frac{d\varphi}{dy}\frac{dy}{d\xi} + \frac{d\varphi}{dz}\frac{dz}{d\xi}\\ &= \frac{d\varphi}{dx}\alpha + \frac{d\varphi}{dy}\alpha_1 + \frac{d\varphi}{dz}\alpha_2\end{aligned}$$

With similar expressions of $\frac{d\varphi}{dy}$ and $\frac{d\varphi}{d\zeta}$.

We have since the axes are rectangular

$$\begin{aligned}&\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1 \qquad \beta_1\gamma_1 + \beta_2\gamma_2 + \beta_3\gamma_3 = 0\\ &\beta_1^2 + \beta_2^2 + \beta_3^2 = 1 \qquad \gamma_1\alpha_1 + \gamma_2\alpha_2 + \gamma_3\alpha_3 = 0\\ &\gamma_1^2 + \gamma_2^2 + \gamma_3^2 = 1 \qquad \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 = 0\\ &\text{\&c.} \qquad\qquad\qquad \text{\&c.}\end{aligned}$$

By observing these relations we obtain from the above equations

$$\left(\frac{d\varphi}{d\xi}\right)^2 + \left(\frac{d\varphi}{d\eta}\right)^2 + \left(\frac{d\varphi}{d\zeta}\right)^2 = \left(\frac{d\varphi}{dx}\right)^2 + \left(\frac{d\varphi}{dy}\right)^2 + \left(\frac{d\varphi}{dz}\right)^2$$

We have again

$$\begin{aligned}\frac{d^2\varphi}{d\xi^2} &= \frac{d^2\varphi}{dx^2}\alpha_1^2 + \frac{d^2\varphi}{dy^2}\alpha_2^2 + \frac{d^2\varphi}{dz^2}\alpha_3^2\\ \frac{d^2\varphi}{d\eta^2} &= \frac{d^2\varphi}{dx^2}\beta_1^2 + \frac{d^2\varphi}{dy^2}\beta_2^2 + \frac{d^2\varphi}{dz^2}\beta_3^2\\ \frac{d^2\varphi}{d\zeta^2} &= \frac{d^2\varphi}{dx^2}\gamma_1^2 + \frac{d^2\varphi}{dy^2}\gamma_2^2 + \frac{d^2\varphi}{dz^2}\gamma_3^2\end{aligned}$$

and consequently

$$\Delta^2_{(x,y,z,)}\varphi = \Delta^2_{(\xi,\eta,\zeta,)}\varphi = 0.$$

Now let u, v, w denote the component velocities along the axes of x, y, z, respectively of the origin of these axes, also p, q, r, the components of angular velocity of the body with reference to the same axes. Then for the components of the total velocity of any point x, y, z of the body in the directions of x, y, z respectively, or what is the same thing for the components of the velocity of any fluid particle that is at rest relatively to the body, we have

$$\begin{aligned}&u + zq - yr\\ &v + xr - zp\\ &w + yp - xq\end{aligned}$$

If the body were at rest and the fluid particle in motion or *vice versa*, the components of the velocity of this particle would be given by,

$$\frac{d\varphi}{dx}, \quad \frac{d\varphi}{dy}, \quad \frac{d\varphi}{dz}$$

So we have for the motion of the fluid particles relatively to the body

$$\frac{dx}{dt}=\frac{d\varphi}{dx}-u-zq+yr$$
$$\frac{dy}{dt}=\frac{d\varphi}{dy}-v-xr+zp$$
$$\frac{dz}{dt}=\frac{d\varphi}{dz}-w-yp+xq$$

To determine the absolute motion of the fluid particle, that is, the motion relatively to the axes of ξ,η,ζ, it is of course necessary to transform these equations by means of the ordinary formulæ, so that ξ,η,ζ, shall become the independent variables, then integration will determine the motion of the particle.

It is necessary now to determine φ subject to the conditions that have been frequently referred to, viz: that it shall be single valued and continuous, that throughout the entire fluid mass $\Delta^2\varphi$ shall $=o$, that φ with its derivatives $\frac{d\varphi}{dx}$, $\frac{d\varphi}{dy}$, $\frac{d\varphi}{dz}$ shall vanish at infinity. It is easy to see that all the conditions will be satisfied. By assuming φ in the form

$$\varphi=u\varphi_1+v\varphi_2+w\varphi_3+p\varphi_4+q\varphi_5+r\varphi_6$$

We have for the velocity of any point of the surface of the body in the direction of the outer normal to the body $\frac{d\varphi}{d\nu}$; but the components of this velocity parallel to the axes of x,y and z are

$$u+zq-yr$$
$$v+xr-zp$$
$$w+yp-xq$$

and consequently for the normal velocity

$$\frac{d\varphi}{d\nu}=p[y\cos(\nu z)-z\cos(\nu y)]+q[z\cos(\nu x)-x\cos(\nu z)]+r[x\cos(\nu y)-y\cos(\nu x)]+u\cos(\nu x)+v\cos(\nu y)+w\cos(\nu z)$$

Then at the surface of the body we must find the following conditions to hold

$$\frac{d\varphi_1}{d\nu}=\cos(\nu x),\quad \frac{d\varphi_4}{d\nu}=y\cos(\nu z)-z\cos(\nu y),$$
$$\frac{d\varphi_2}{d\nu}=\cos(\nu y),\quad \frac{d\varphi_5}{d\nu}=z\cos(\nu x)-x\cos(\nu z)$$
$$\frac{d\varphi_3}{d\nu}=\cos(\nu z),\quad \frac{d\varphi_6}{d\nu}=x\cos(\nu y)-y\cos(\nu x).$$

These equations enable us to determine the functions $\varphi_1 \ldots \varphi_6$ which are evidently dependent *only on the form of the body*. Take first the case when the body is an ellipsoid; we found for that

$$\Phi=\text{const.}-\pi(Ax^2+By^2+Cz^2)$$

and that

$$G\,\frac{d^2\Phi}{d\nu dz}=\cos(\nu z)$$

when G was found to be $=2\pi(2-C)$; thus we have

$$\frac{d^2\Phi}{d\nu dx}=2\pi(2-A)\cos(\nu x)$$
$$\frac{d^2\Phi}{d\nu dy}=2\pi(2-B)\cos(\nu y)$$
$$\frac{d^2\Phi}{d\nu dz}=2\pi(2-C)\cos(\nu z)$$

Substituting for $\cos(\nu,x)$, $\cos(\nu,y)$, $\cos(\nu z)$ their values as given above

$$\frac{1}{2\pi(2-A)}\,\frac{d^2\Phi}{d\nu dx}=\frac{d\varphi_1}{d\nu}$$
$$\frac{1}{2\pi(2-B)}\,\frac{d^2\Phi}{d\nu dy}=\frac{d\varphi_2}{d\nu}$$
$$\frac{1}{2\pi(2-C)}\,\frac{d^2\Phi}{d\nu dz}=\frac{d\varphi_3}{d\nu}$$

Integrating

$$\varphi_1=\frac{1}{2\pi(2-A)}\,\frac{d\Phi}{dx}$$
$$\varphi_2=\frac{1}{2\pi(2-B)}\,\frac{d\varphi}{dy}$$
$$\varphi_3=\frac{1}{2\pi(2-C)}\,\frac{d\Phi}{dz}$$

If we write for $\frac{d\Phi}{dx}$ . . . their values of $-2\pi Ax$ . . . these can be written

$$\varphi_1=\frac{A}{A-2}\,x$$
$$\varphi_2=\frac{B}{B-2}\,y$$
$$\varphi_3=\frac{C}{C-2}\,z$$

Before determining the quantities φ_4, φ_5 and φ_6 it is worthy of notice that for a sphere we have

$$\frac{d\varphi_4}{d\nu}=\frac{d\varphi_5}{d\nu}=\frac{d\varphi_6}{d\nu}=o$$

that is, the normal velocity of a particle

at the surface of the sphere is independent of any motion of rotation that the sphere may possess; we shall see that this is true for any particle of the fluid mass by showing that $\varphi_4=\varphi_5=\varphi_6=o$.

We have as is obvious

$$\frac{d\varphi_4}{d\nu}=y\frac{d\varphi_3}{d\nu}-z\frac{d\varphi_2}{d\nu}$$
$$\frac{d\varphi_5}{d\nu}=z\frac{d\varphi_1}{d\nu}-x\frac{d\varphi_3}{d\nu}$$
$$\frac{d\varphi_6}{d\nu}=x\frac{d\varphi_2}{d\nu}-y\frac{d\varphi_1}{d\nu}$$

The forms of φ_4, φ_5, φ_6 give us a clue to the possible forms of φ_4, φ_5 and φ_6. At all events we can avail ourselves of the analogy and see if we can find constants H_1, H_2 and H_3 such that we shall have

$$\varphi_4=H_1\left(y\frac{d\Phi}{dz}-z\frac{d\Phi}{dy}\right)$$
$$\varphi_5=H_2\left(z\frac{d\Phi}{dx}-x\frac{d\Phi}{dz}\right)$$
$$\varphi_6=H_3\left(x\frac{d\Phi}{dy}-y\frac{d\Phi}{dx}\right)$$

Arbitrary constants will satisfy all the conditions imposed on these quantities φ_4, φ_5, φ_6, but it will be necessary to find certain definite values for them in order to satisfy the surface conditions. Now we know that

$$\frac{dx}{d\nu}=\cos(\nu x),\quad \frac{dy}{d\nu}=\cos(\nu y)$$

and also

$$\frac{d\Phi}{dx}=-2\pi Ax\quad \frac{d\Phi}{dy}=-2\pi By.$$
$$\frac{d^2\Phi}{d\nu dx}=2\pi(z-A)\cos\nu x,$$
$$\frac{d^2\Phi}{d\nu dy}=2\pi(z-B)\cos\nu y.$$

Differentiating φ_6, and substituting these values

$$\frac{d\varphi_6}{d\nu}=H_3\left\{\frac{d\Phi}{dy}\cos\nu x+x2\pi(z-B\cos\nu y\right.$$
$$\left.-\frac{d\Phi}{dx}\cos\nu y-y\,2\pi(z-A)\cos\nu x\right\}$$

therefore

$$x\cos(\nu y)-y\cos(\nu x)$$
$$=2\pi H_3[x\cos(\nu y)[2+(A+B)]$$
$$-y\cos(\nu x)[2+(B-A)]]$$

For the ellipsoid we have, however,

$$\cos(\nu x):\cos(\nu y):\cos(\nu z)=\frac{x}{a^2}:\frac{y}{b^2}:\frac{z}{c^2}$$

or

$$x=h\,.\,a^2\cos(\nu x)$$
$$y=h\,.\,b^2\cos(\nu y)$$
$$z=h\,.\,c^2\cos(\nu z)$$

Substituting these values for x and y in both sides of the above equations we find the common factor $\cos(\nu x)\cos(\nu y)$ on omitting which, the equation gives us

$$H_3=\frac{a^2-b^2}{2\pi[z(a^2-b^2)+(A-B)(a^2+b^2)]}$$

and in like manner by simply advancing the letters

$$H_1=\frac{b^2-c^2}{2\pi[2(b^2-c^2)-(B-C)(b^2+c^2)]}$$
$$H_2=\frac{c^2-a^2}{2\pi[2(c^2-a^2)-(C-A)(c^2+a^2)]}$$

If we have either $a=b$ or $b=c$, that is, if our ellipsoid becomes either of the two ellipsoids of revolution, the constants H_1 and H_3 have finite values, since $\frac{A-B}{a^2-b^2}$ will be finite in the limit as elsewhere. The quantities

$$x\frac{d\Phi}{dy}-y\frac{d\Phi}{dx}$$

and

$$y\frac{d\Phi}{dz}-z\frac{d\Phi}{dy}$$

will, however, vanish in these cases; thus giving us that a rotation about the axis of revolution of either of the ellipsoids of revolution has no effect upon the motion of the fluid. Further, as the coefficients of all three of the finite quantities H_1, H_2, H_3 vanish in the case of the sphere, we have that no motion of rotation of the sphere produces any effect upon the motion of the fluid. We know the value of Φ for the sphere, the volume of the sphere divided by the distance from its center to the point of the fluid under consideration, and also for the sphere we have $A=B=C=\frac{2}{3}$, consequently

$$\varphi_1=\frac{R^3}{2}\frac{d}{dx}\cdot\frac{1}{r}\quad \varphi_2=\frac{R^3}{2}\frac{d}{dy}\cdot\frac{1}{r}\cdot$$
$$\varphi_3=\frac{R^3}{2}\cdot\frac{d}{dz}\cdot\frac{1}{r}$$

and therefore

$$\varphi=\frac{R^3}{2}\left\{u\frac{d}{dx}\cdot\frac{1}{r}+v\frac{d}{dy}\cdot\frac{1}{r}+w\frac{d}{dz}\cdot\frac{1}{r}\right\}$$

Now we have $\frac{d\frac{1}{r}}{dr}=-\frac{1}{r^2}$ and $\frac{d^2}{dxdr}\cdot\frac{1}{r}=-\frac{2x}{r^4}$; at the surface of the sphere R=r and we have then

$$\frac{d\varphi}{d\nu}=-\frac{d\varphi}{dR}=u\frac{x}{r}+v\frac{y}{r}+w\frac{z}{r}$$

$$\frac{d\varphi}{d\nu}=u\cos(\nu x)+v\cos(\nu y)+w\cos(\nu z)$$

the condition at the surface of the sphere. If we take as one body a ring whose equation is

$$(r\pm a)^2+z^2=c^2$$

when $r=\sqrt{x^2+y^2}$ and the vectorial angle is θ, we have, as we shall see hereafter, for its potential at an external point

$$\Phi=r-\tfrac{1}{2}U$$

when U is given by the series

$$U=P_0^{0,0}+P^{1,1}\begin{matrix}\cos\\\sin\end{matrix}\psi\begin{matrix}\cos\\\sin\end{matrix}\theta+P^{2,2}\begin{matrix}\sin\\\cos\end{matrix}\psi\begin{matrix}\sin\\\cos\end{matrix}2\theta+$$

The general term being

$$P^{n,m}\begin{matrix}\cos\\\sin\end{matrix}n\psi\begin{matrix}\cos\\\sin\end{matrix}m\theta$$

the quantities P and ψ do not depend on x, y, z so that we shall have here again

$$x\frac{d\Phi}{dy}-y\frac{d\Phi}{dx}=o$$

or if the ring simply rotate around its axis it will produce no effect upon the motions of the fluid particles.

The quantity which we have denoted by T, that is, the energy of the body is, as we know, a homogeneous function of the second degree of the quantities u,v,w,p,q,r, and is positive. We have, when no external forces act on the body, the following expressions out of Analytic Mechanics divided from T:

$$\frac{d}{dt}\cdot\frac{dT}{du}=r\frac{dT}{dv}-q\frac{dT}{dw}$$

$$\frac{d}{dt}\cdot\frac{dT}{dv}=p\frac{dT}{dw}-r\frac{dT}{du}$$

$$\frac{d}{dt}\cdot\frac{dT}{dw}=q\frac{dT}{du}-p\frac{dT}{dv}$$

$$\frac{d}{dt}\cdot\frac{dT}{dp}=w\frac{dT}{dv}-v\frac{dT}{dw}+r\frac{dT}{dq}-q\frac{dT}{dr}$$

$$\frac{d}{dt}\cdot\frac{dT}{dq}=u\frac{dT}{dw}-w\frac{dT}{du}+p\frac{dT}{dr}-r\frac{dT}{dp}$$

$$\frac{d}{dt}\cdot\frac{dT}{dr}=v\frac{dT}{du}-u\frac{dT}{dv}+q\frac{dT}{dp}-p\frac{dT}{dq}$$

If we assume that the body has no motion of rotation, that is $p=q=r=o$, and that u, v, w are constants fulfilling the conditions

$$u:v:w=\frac{dT}{du}:\frac{dT}{dv}:\frac{dT}{dw}$$

the above equations will be satisfied, both sides being $=o$. But if p, q, r are equal to zero then T will be a homogeneous function of the second degree in u, v, w, and as all its terms are positive the equation

$$T=\text{const.}$$

must be that of an ellipsoid, the co-ordinates of any point on which are u, v, w. The determination of the ratios $u:v:w$ will now coincide with the determination of the principal axes of this ellipsoid. If we take u, v, w, parallel to x, y, z then the then principal axes of the ellipsoid will coincide in direction with three fixed directions x,y,z in the body, in any one of which the body may move with constant velocities respectively of u, v, w and without any motion of rotation. If the ellipsoid be of rotation the body can move in the direction of the axes of rotation or any direction perpendicular to that direction and have no motion of rotation. A sphere can move in any direction without having produced in it a motion of rotation. Since T is a homogeneous factor of the second degree in u, v, w, p, q, r, we have by Euler's Theorem

$$2T=u\frac{dT}{du}+v\frac{dT}{dv}+w\frac{dT}{dw}+p\frac{dT}{dp}+q\frac{dT}{dq}+r\frac{dT}{dr}$$

and again as T, u, v . . . r are functions of t we have

$$\frac{dT}{dt}=\frac{dT}{du}\frac{du}{dt}+\frac{dT}{dv}\frac{dv}{dt}+\cdots\frac{dT}{dr}\frac{dr}{dt}$$

but from the last equation we obtain

$$2\frac{d\mathrm{T}}{dt}=\frac{du}{dt}\frac{d\mathrm{T}}{du}+\frac{dv}{dt}\frac{d\mathrm{T}}{dv}+\ldots\frac{dr}{dt}\frac{d\mathrm{T}}{dr}$$
$$+u\frac{d}{dt}\cdot\frac{d\mathrm{T}}{du}+v\frac{d}{dt}\cdot\frac{d\mathrm{T}}{dv}+\ldots r\frac{d}{dt}\cdot\frac{d\mathrm{T}}{dr}$$

combination of the last two equations gives

$$\frac{d\mathrm{T}}{dt}=u\frac{d}{dt}\cdot\frac{d\mathrm{T}}{du}+v\frac{d}{dt}\frac{d\mathrm{T}}{dv}$$
$$+w\frac{d}{dt}\cdot\frac{d\mathrm{T}}{dw}+\ldots r\frac{d}{dt}\frac{d\mathrm{T}}{dr}$$

Now if we multiply the equations of motion as above written by u, v, w, p, q, r, respectively, and add them together we have, by observing the last result $\frac{d\mathrm{T}}{dt}=o$

or $$2\mathrm{T}=\mathrm{F}' \text{ a const.}$$

Again multiply the first three of the equations respectively by $\frac{d\mathrm{T}}{du}, \frac{d\mathrm{T}}{dv}, \frac{d\mathrm{T}}{dw}$, and add, we have then

$$\frac{d}{dt}\left\{\left(\frac{d\mathrm{T}}{du}\right)^2+\left(\frac{d\mathrm{T}}{dv}\right)^2+\left(\frac{d\mathrm{T}}{dw}\right)^2\right\}=o$$

or

$$\left(\frac{d\mathrm{T}}{du}\right)^2+\left(\frac{d\mathrm{T}}{dv}\right)^2+\left(\frac{d\mathrm{T}}{dw}\right)^2=\mathrm{E}'\text{a const.}$$

And finally multiplying the entire set of equations by $\frac{d\mathrm{T}}{du}, \frac{d\mathrm{T}}{dv}, \frac{d\mathrm{T}}{dw}, \frac{d\mathrm{T}}{dp}, \frac{d\mathrm{T}}{dq}, \frac{d\mathrm{T}}{dr}$, respectively, and adding we have again after integration

$$\frac{d\mathrm{T}}{du}\cdot\frac{d\mathrm{T}}{dp}+\frac{d\mathrm{T}}{dv}\cdot\frac{d\mathrm{T}}{dq}+\frac{d\mathrm{T}}{dw}\cdot\frac{d\mathrm{T}}{dr}=\mathrm{G}' \text{ a const.}$$

Transformation to the fixed axes ξ,η,ζ, will give us six more integrals. The new forms of the equations of motion are, after integration with respect to t,

$$\alpha_1\frac{d\mathrm{T}}{du}+\alpha_2\frac{d\mathrm{T}}{dv}+\alpha_3\frac{d\mathrm{T}}{dw}=\mathrm{A}'$$
$$\beta_1\frac{d\mathrm{T}}{du}+\beta_2\frac{d\mathrm{T}}{dv}+\beta_3\frac{d\mathrm{T}}{dw}=\mathrm{B}'$$
$$\gamma_1\frac{d\mathrm{T}}{du}+\gamma_2\frac{d\mathrm{T}}{dv}+\gamma_3\frac{d\mathrm{T}}{dw}=\mathrm{C}'$$

$$\alpha_1\frac{d\mathrm{T}}{dp}+\alpha_2\frac{d\mathrm{T}}{dq}+\alpha_3\frac{d\mathrm{T}}{dr}=\mathrm{A}''+\mathrm{B}'\gamma-\mathrm{C}'\beta$$
$$\beta_1\frac{d\mathrm{T}}{dp}+\beta_2\frac{d\mathrm{T}}{dq}+\beta_3\frac{d\mathrm{T}}{dr}=\mathrm{B}''+\mathrm{C}'\alpha-\mathrm{A}'\gamma$$
$$\gamma_1\frac{d\mathrm{T}}{dp}+\gamma_2\frac{d\mathrm{T}}{dq}+\gamma_3\frac{d\mathrm{T}}{dr}=\mathrm{C}''+\mathrm{A}'\beta-\mathrm{B}'\alpha$$

$\mathrm{A}', \mathrm{B}', \ldots \mathrm{C}''$ being arbitrary constants. Squaring and adding together the first three of these, we have, by virtue of the relations among the direction cosines

$$\mathrm{A}'^2+\mathrm{B}'^2+\mathrm{C}'^2=\mathrm{E}'$$

and again without difficulty

$$\mathrm{A}'\mathrm{A}''+\mathrm{B}'\mathrm{B}''+\mathrm{C}'\mathrm{C}''=\mathrm{G}'$$

The motion of the body is completely known if the quantities $\alpha \ldots \gamma_3$ which determine its position at any time t. These can be determined now as will be seen if the quantities $u, v \ldots r$ can be assigned as certain definite functions of t. If we regard $\frac{d\mathrm{T}}{du}, \frac{d\mathrm{T}}{dv}, \frac{d\mathrm{T}}{dw}$ as the components of the velocity of a point in the directions of the axes x, y, z, respectively, then the quantities $\mathrm{A}', \mathrm{B}', \mathrm{C}'$, will be the components of these velocities in the directions of ξ, η, ζ respectively; now if we take the axes of ζ in the direction of the resultant of the velocities, we will have necessarily

$$\mathrm{A}'=\mathrm{B}'=o.$$

This being a perfectly legitimate transformation, we will now assume $\mathrm{A}'=\mathrm{B}'=o$. Multiply the equations for $\mathrm{A}', \mathrm{B}', \mathrm{C}'$ by $\alpha_1, \beta_1, \gamma_1$ then by $\alpha_2, \beta_2, \gamma_2$, and finally by $\alpha_3, \beta_3, \gamma_3$, and add the separate results; we have thus

$$r_1=\frac{1}{\mathrm{C}'}\frac{d\mathrm{T}}{du}, \quad r_2=\frac{1}{\mathrm{C}'}\frac{d\mathrm{T}}{dv}, \quad r_3=\frac{1}{\mathrm{C}'}\frac{d\mathrm{T}}{dw}$$

If we introduce the polar angles ϑ, ψ, φ by means of the equations

$$\alpha_1=-\cos\varphi\cos\psi\cos\vartheta-\sin\varphi\sin\psi,$$
$$\beta=-\sin\varphi\cos\psi\cos\vartheta+\cos\varphi\sin\psi,$$
$$\gamma_1=\cos\psi\sin\vartheta$$
$$\alpha_2=-\cos\varphi\sin\psi\cos\vartheta+\sin\varphi\cos\psi,$$
$$\beta_2=-\sin\varphi\sin\psi\cos\vartheta-\cos\varphi\cos\psi,$$
$$\gamma_2=\sin\psi\sin\vartheta$$
$$\alpha_3=\cos\varphi\sin\vartheta,$$
$$\beta_3=\sin\varphi\sin\vartheta,$$
$$\gamma_3=\cos\vartheta.$$

The expressions for γ_1 and γ_2 give for ψ

$$\tan\psi=\frac{r_2}{r_1}$$

therefore

$$\psi=\tan^{-1}\left\{\frac{d\mathrm{T}}{dv}\div\frac{d\mathrm{T}}{du}\right\}$$

and also

$$\Im = \cos^{-1}\left\{\frac{1}{C'}\frac{dT}{dw}\right\}$$

From the expressions for β_3 and a_3 we have

$\tan\varphi = \frac{\beta_3}{a_3}$ which gives $\cos^2\varphi = \frac{a_3^2}{1-\gamma_3^2}$

By virtue of the relation $a_3^2+\beta_3^2+\gamma_3^2=1$

Differentiating this and solving for $d\varphi$ and we obtain

$$d\varphi = \frac{a_3 d\beta_3 - \beta_3 da_3}{1-\gamma_3^2}$$

But we have for $d\beta_3$ and da_3 certain known values, viz:

$d\beta_3=(\beta_1 q-\beta_2 p)dt$, $da_3=(a_1 q-a_2 p)dt$

consequently

$$d\varphi = \frac{r_1 p + r_2 q}{r_1^2+r_2^2}\,dt$$

which on substituting the values of r_1 and r_2 becomes

$$d\varphi = C'.\frac{p\frac{dT}{du}+q\frac{dT}{dv}}{\left(\frac{dT}{du}\right)^2+\left(\frac{dT}{dv}\right)^2}dt$$

We can obtain another value for ψ which is worthy of notice. We had

$\tan\psi = \frac{r_2}{r_1}$ which gives $\cos^2\psi = \frac{r_1^2}{r_1^2+r_2^2}$

Since $r_1^2+r_2^2+r_3^2=1$; differentiating this and solving for $d\psi$ we get

$$d\psi = \frac{r_1 dr_2 - r_2 dr_1}{r_1^2+r_2^2}$$

but $dr_1=(r_2 r-r_3 q)dt$, $dr_2=(r_3 p-r_1 r)dt$,

consequently

$$d\psi = \frac{r_3(r_1 p+r_2 q)-(r_1^2+r_2^2)r}{r_1^2+r_2^2}\,dt$$

or,

$$d\psi = \cos\Im\, d\varphi - r dt$$

from which we have finally

$$d\psi = \frac{p\frac{dT}{du}\cdot\frac{dT}{dw}+q\frac{dT}{dv}\cdot\frac{dT}{dw}-r\left\{\left(\frac{dT}{du}\right)^2+\left(\frac{dT}{dv}\right)^2\right\}}{\left(\frac{dT}{du}\right)^2+\left(\frac{dT}{dv}\right)^2}dt$$

This can be obtained by direct differentiation of the previous value given for ψ. This gives

$$\frac{d\psi}{dt} = \frac{\frac{dT}{du}\cdot\frac{d}{dt}\cdot\frac{dT}{dv}-\frac{dT}{dv}\cdot\frac{d}{dt}\cdot\frac{dT}{du}}{\left(\frac{dT}{dv}\div\frac{dT}{du}\right)^2 \div\left\{1+\left(\frac{dT}{dv}\div\frac{dT}{du}\right)^2\right\}}$$

$$= \frac{\frac{dT}{du}\cdot\frac{d}{dt}\cdot\frac{dT}{dv}-\frac{dT}{dv}\cdot\frac{d}{dt}\cdot\frac{dT}{du}}{\left(\frac{dT}{du}\right)^2+\left(\frac{dT}{dv}\right)^2}$$

Substituting for $\frac{d}{dt}\cdot\frac{dT}{du}$, and $\frac{d}{dt}\cdot\frac{dT}{dv}$ their values from the equations of motion of the body and we obtain the result given above. We have now the means of determining the motion of the body if the origin of x, y, z remains stationary. To determine the completely general motion then, we have as yet to find the values of the quantities α, β, γ which gives the position of the origin, as functions of the time. We can assume without any loss of generality that $A''=B''=o$. Then, since we had $A'=B'=o$, we have from the equations which gave A'' and B''

$$\dot{\alpha} = \frac{1}{C'}\left(\beta_1\frac{dT}{dp}+\beta_2\frac{dT}{dq}+\beta_3\frac{dT}{dr}\right)$$

$$\beta = -\frac{1}{C'}\left(a_1\frac{dT}{dp}+a_2\frac{dT}{dq}+\dot{a}_3\frac{dT}{dr}\right)$$

For r we have since $\frac{dr}{dt}$ is the velocity of the origin of xyz in the direction of δ,

$$\frac{dr}{dt} = r_1 u + r_2 v + r_3 w$$

From which by giving r_1, r_2, r_3 their values as above found

$$\frac{dr}{dt} = \frac{1}{C'}\left(u\frac{dT}{du}+v\frac{dT}{dv}+w\frac{dT}{dw}\right)$$

There are certain cases when all of our results are much simplified by certain simplifications that T may receive. The quantity denoted by T is the energy of the entire system. This is divided into two parts, that due to the fluid, or calling ρ its density,

$$T' = \frac{\rho}{2}\int d\tau\left\{\left(\frac{d\varphi}{dx}\right)^2+\left(\frac{d\varphi}{dy}\right)^2+\left(\frac{d\varphi}{dz}\right)^2\right\}$$

when φ is the velocity potential: second that due to the body

$$2T''=fdm[(u+zq-yr)^2+(v+xr-zp)^2+(w+yp-xq)^2]$$

We may if we choose write for these two expressions two other more general ones which contain the remaining quantities of the second order which can be formed out of the quantities $u, v, \ldots r$. Thus for twice the energy of the fluids write

$$\begin{aligned}2T'=&a_{11}u^2+2a_{12}uv+2a_{13}uw+2a_{14}up+\ldots\\&+a_{22}v^2+2a_{23}vw+2a_{24}yp+\ldots\\&+a_{33}w^2+2a_{34}wp+\ldots\\&\vdots\quad\vdots\quad\vdots\end{aligned}$$

and for twice that of the body

$$\begin{aligned}2T''=&b_{11}u^2+2b_{12}uv+2b_{13}uw+2b_{14}up+\ldots\\&+b_{22}v^2+2b_{23}vw+2b_{24}vp+\ldots\\&+b_{33}w^2+2b_{34}wp+\ldots,\\&\vdots\quad\vdots\quad\vdots\end{aligned}$$

the sum of these or twice the energy of the sytem we may write

$$\begin{aligned}2T=&c_{11}u^2+2c_{12}uv+2c_{13}uw+2c_{14}up+\ldots\\&+c_{22}v^2+2c_{23}vw+2c_{24}vp+\ldots\\&\vdots\quad\vdots\quad\vdots\quad\vdots\end{aligned}$$

We have in each of these expressions 21 constants. But only fifteen of these exist in reality as none of the terms uv, uw, vw, up, vq, wr, have any existence in the expressions for the energy. Suppose the body to be symmetrical with respect to one of the co-ordinate planes, say xz, then there are two points (x, y, z) and $(x, -y, z)$ either of which being substituted in the expression for the energy must leave this quantity unaltered. Or what is the same thing, we may reverse the direction of the axis of y and T'' must remain unaltered. We see from the integral expression for T'' that the quantities

$$wp-ur,\ qr,\ pq$$

are multiplied by y they will therefore change signs and consequently their coefficients $b_{24}, b_{12}, b_{23}, b_{14}$ are equal to zero. Also we have as above remarked

$$b_{12}=b_{14}=b_{13}=b_{36}=b_{25}=b=o$$

or finally if the body is such that we can change y into $-y$ we will have

$$\begin{aligned}b_{12}=b_{14}=b_{16}=b_{22}=b_{24}=b_{26}\\=b_{32}=b_{34}=b=_{50}=o\end{aligned}$$

If we can change z, into $-z$, we shall have

$$b_{29}=b_{25}=b_{24}=b_{16}=b_{15}=b_{13}=b_{23}=b_{63}=b_{64}=o$$

and finally if we can reverse x

$$\begin{aligned}b_{31}=b_{36}=b_{35}=b_{21}=b_{26}=b_{25}\\=b_{41}=b_{46}=b_{45}=o\end{aligned}$$

With respect to T′ the energy of the fluid similar results are obtained, but necessarily in a different manner. We had

$$\varphi=u\varphi_1+v\varphi_2+w\varphi_3+p\varphi_4+q\varphi_5+r\varphi_6$$

also

$$\begin{aligned}\frac{d\varphi}{d\nu}=&u\frac{d\varphi_1}{d\nu}+v\frac{d\varphi_2}{d\nu}+w\frac{d\varphi_3}{d\nu}\\&+p\frac{d\varphi_4}{d\nu}+q\frac{d\varphi_5}{d\nu}+r\frac{d\varphi_6}{d\nu}\end{aligned}$$

By partial integration of

$$\frac{\rho}{2}\int d\tau\left\{\left(\frac{d\varphi}{dx}\right)^2+\left(\frac{d\varphi}{dy}\right)^2+\left(\frac{d\varphi}{dz}\right)^2\right\}$$

and observing the above relations we can write it as

$$-\frac{\rho}{2}\int d\sigma\varphi\frac{d\varphi}{d\nu}$$

the integration being extended over the surface of an infinitely great sphere which we may suppose to contain the fluid. Now from the values given of $\frac{d\varphi_1}{d\nu}\ldots\frac{d\varphi_6}{d\nu}$ we see that for a reversal of the axis of y that $\frac{d\varphi_1}{d\nu}, \frac{d\varphi_3}{d\nu}, \frac{d\varphi_5}{d\nu}$ do not alter but $\frac{d\varphi_2}{d\nu}, \frac{d\varphi_4}{d\nu}, \frac{d\varphi_6}{d\nu}$ change signs, and we can obtain from this fact that $\varphi_1, \varphi_3, \varphi_5$ do not change, but that $\varphi_2, \varphi_4, \varphi_6$ do change their signs by reversing the direction of y. Now we had by virtue of the above expression for the energy

$$a_{11}=-\rho\int d\sigma\varphi_1\frac{d\varphi_1}{d\nu},$$

$$a_{12}=-\rho\int d\sigma\varphi_1\frac{d\varphi_2}{d\nu}=-\rho\int d\sigma\varphi_2\frac{d\varphi}{d\nu}$$

. . . . . .

Take for example a_{12}: this integral has at the points (x,y,z) and $(x,-y,z)$ opposite values, and consequently $a_{12}=o$ and we have as before

$$\begin{aligned}a_{12}=a_{14}=a_{16}=a_{32}=a_{56}=a_{34}=a_{61}=a_{63}\\=a_{56}=o\end{aligned}$$

and similar expressions if we can reverse

z or x. Thus finally we have for the energy of the system in the case when there is symmetry with respect to the plane xz

$$C_{(135)(246)}=o$$

and for symmetry with respect to yx and zy respectively,

$$C_{(216)(354)}=o,\ C_{(324)(165)}=o.$$

If the body be symmetrical with respect to both of the planes xz and yz we shall have for the energy

$$2T=C_{11}u^2+C_{22}v^2+C_{33}w^2+C_{44}p^2 + C_{55}q^2+C_{66}r^2+zC_{15}uq+zC_{24}vp$$

We have thus reduced the number of the coefficients to 8. Now suppose that we can rotate the axes of x and y about z through an angle ϑ leaving T unchanged.

The formulæ for the rotation are, as is well known,

$$\begin{aligned} x&= x'\cos\vartheta+y'\sin\vartheta\\ y&=-x'\sin\vartheta+y'\cos\vartheta\\ z&=-z' \end{aligned}$$

we have also

$$\begin{aligned} u&= u'\cos\vartheta+v'\sin\vartheta, & p&= p'\cos\vartheta+q'\sin\vartheta\\ v&=-u'\sin\vartheta+v'\cos\vartheta, & q&=-p'\sin\vartheta+q'\cos\vartheta\\ w&=w' & r&=\sigma' \end{aligned}$$

Using the accented letters gives us

$$2T=C_{11}'u'^2+C_{22}'v'^2+C_{33}'w'^2+C_{44}'p'^2 + C_{55}'q'^2+C_{66}'r'^2+2C_{15}'u'q'+2C_{24}'v'p'$$

Substituting in the previous value of T for u,v, and w their values in terms of u',v',w', and similarly for p,q,r, we have

$$\begin{aligned} 2T=&u'^2[C_{11}\cos^2\vartheta+C_{22}\sin^2\vartheta] + v'^2[C_{11}\sin^2\vartheta+C_{22}\cos^2\vartheta]+w'^2C_{33}\\ &+p'^2[C_{44}\cos^2\vartheta+C_{55}\sin^2\vartheta] + q'^2[C_{44}\sin y+C_{55}\cos^2\vartheta]+r'^2C_{66}\\ &+u'v'[(C_{11}-C_{22})\cos\vartheta\sin\vartheta] + p'q'[(C_{44}-C_{55})\cos\vartheta\sin\vartheta]\\ &+2u'q'[C_{15}\cos^2\vartheta-C_{24}\sin^2\vartheta] + 2v'p'[C_{24}\cos^2\vartheta-C_{15}\sin^2\vartheta]\\ &+2(v'q'-u'p')[(C_{15}+C_{24})\cos\vartheta\sin\vartheta] \end{aligned}$$

Equating to zero the coefficients of $u'v'$, $p'q'$, and $v'q'-u'p'$ since these cannot enter into the value of T, and we have

$$C_{11}=C_{22}\quad C_{44}=C_{55}\quad C_{15}=-C_{24}$$

Now the two expressions for T are identically equal, consequently

$$\begin{aligned} &C_{11}=C_{22}=C_{11}'=C_{22}',C_{44}=C_{55}=C_{44}'=C_{55}',\\ &C_{66}=C_{66}'C_{15}=C_{15}'=-C_{24}=-C_{24}'C_{33}=C_{33}' \end{aligned}$$

Thus we have for T

$$2T=C_{11}(u^2+v^2)+C_{33}w^2+C_{44}(p^2+q^2) + C_{66}r^2+2C_{15}(uq-vp).$$

Suppose now that the axes remaining parallel, we slide the origin along the axes of z, giving us

$$x=x',\ y=y',\ z=z'+a,$$

and then

$$u=u'-ay',\ v=v'+ap',\ w=w',\quad p=p',\ q=q',\ r=r',$$

and that we have

$$\begin{aligned} &C_{11}(u^2+v^2)+C_{33}w^2+C_{44}(p^2+q^2) + C_{66}r^2+2C_{15}(uq-vp)\\ &C_{11}'(u'^2+v'^2)+C_{33}'w'^2+C_{44}(p'^2+q^2) + C_{66}'r'^2+2C_{15}'(u'q'-v'p') \end{aligned}$$

the same process that we pursued before will now give us

$$\begin{aligned} C_{11}&=C_{11}',\ C_{33}=C_{33}',\ C_{66}=C'_{66}\\ C_{44}&=C_{44}'+2aC_{15}'+a^2C_{11}'\\ C_{15}&=C_{15}'+2aC_{11}' \end{aligned}$$

Now from this last we can clearly have by a proper determination of a, that is of the origin

$$C_{15}=o$$

and we shall then have for the energy

$$2T=C_{11}(u^2+v^2)+C_{33}w^2+C_{44}(p^2+q^2)C_{66}r^2$$

This case that we have finished of symmetry about an axis evidently includes all bodies of rotation, and all right prisms or pyramids having square or regular hexagonal right section. If the body is symmetrical with respect to two axes as a sphere or regular octahedron, it is easy to show that we have

$$2T=C_{11}(u^2+v^2+w^2)C_{44}(p^2+q^2+r^2)$$

In the case of the sphere we have seen that

$$\varphi_1=\frac{R^3}{z}\cdot\frac{d}{dx}\cdot\frac{1}{r}$$

this together with the equation

$$a_{11}=-\rho\int d\sigma\varphi_1\frac{d\varphi_1}{dv}$$

gives us for the increase of mass of the sphere due to the influence of the fluid,

$\frac{\rho}{2}\frac{4\pi}{3}R^3$ = to half the mass of the displaced fluid.

The assumption that the body is one of revolution necessarily simplifies much the equations of motion. If we assume then a solid of revolution or having that character we have for twice the energy of the system

$$2U=C_{11}(u^2+v^2)C_{33}w^2+C_{44}(p^2+q^2)C_{66}r^2$$

Substituting this in the equations of motion and they become

$$C_{11}\frac{du}{dt}=C_{11}vr-C_{33}wq$$

$$C_{11}\frac{dv}{dt}=-C_{11}ur+C_{33}wp$$

$$C_{33}\frac{dw}{dt}=C_{11}(uq-vp)$$

$$C_{44}\frac{dp}{dt}=(C_{11}-C_{33})vw+(C_{44}-C_{66})qr$$

$$C_{44}\frac{dq}{dt}=(C_{33}-C_{11})uw+(C_{66}-C_{44})pr$$

$$\frac{dr}{dt}=o \qquad \text{or } r=\text{const.}$$

Now we had $\frac{r_2}{r_1}=\tan\psi=\frac{v}{u}$ and so we can write $u=\lambda\cos\psi$, $v=\lambda\sin\psi$. We can if we wish now introduce a new angle χ such that

$$\frac{p}{q}=\cot(\psi+\chi)$$

from this we can have

$$p=\lambda\cos(\psi+\chi)$$
$$q=\lambda\sin(\psi+\chi)$$

and as a result of these transformations

$$u^2+v^2=l^2 \quad p^2+q^2=\lambda^2,$$
$$udu+vdv=ldl, \quad pdp+qdq=\lambda d\lambda,$$
$$up+vq=l\lambda\cos\chi, \quad uq-vp=l\lambda\sin\chi,$$
$$l^2d\psi=udv-vdu, \quad \lambda^2(d\psi+d\chi)=pdq-qdp;$$

The first two of our equations now gives us

$$C_{11}\frac{dl}{dt}=-C_{33}w\lambda\sin\chi$$

and

$$C_{11}\frac{d\psi}{dt}=C_{33}w\frac{\lambda}{l}\cos\chi-C_{11}r$$

the third becomes

$$C_{33}\frac{dw}{dt}=C_{11}l\lambda\sin\chi$$

and the fourth and fifth combined

$$C_{44}\frac{d\lambda}{dt}=(C_{33}-C_{11})wl\sin\chi$$

$$C_{44}\frac{d\chi}{dt}=\left\{(C_{33}-C_{11})\frac{l}{\lambda}-\frac{C_{33}C_{44}}{C_{11}}\frac{\lambda}{l}\right\}w\cos\chi+C_{66}r$$

We have already obtained the following integrals:

$$2T=F'$$

$$\left(\frac{dT}{du}\right)^2+\left(\frac{dT}{dv}\right)^2+\left(\frac{dT}{dw}\right)^2=E'$$

$$\frac{dT}{du}\cdot\frac{dT}{dp}+\frac{dT}{dv}\cdot\frac{dT}{dq}+\frac{dT}{dw}\cdot\frac{dT}{dr}=G'$$

The new forms of the first members of these are respectively

$$C_{11}l^2+C_{33}w^2+C_{44}\lambda^2+C_{66}r^2=F'$$
$$C_{11}{}^2l^2+C_{33}{}^2w^2 \qquad =E'$$
$$C_{33}C_{66}wr+C_{11}C_{44}l\lambda\cos\chi \quad =G'$$

Remembering now that r is constant, introduce the following constants:

$$e=\frac{E'}{C_{11}{}^2}, \quad f=\frac{F'-(C_{66}r^2+l)}{C_{44}}, \quad g=\frac{G'}{C_{11}C_{44}}$$

$$e'=\left(\frac{C_{33}}{C_{11}}\right)^2, \quad f'=\frac{C_{33}-C_{11}C'}{C_{44}}, \quad g'=\frac{C_{33}C_{66}r}{C_{11}C_{44}}$$

Thus we can now have for the quantities l,λ, and $l\lambda\cos\chi$ the following:

$$l=\sqrt{e-e'w^2}$$
$$\lambda=\sqrt{f-f'w^2}$$
$$l\lambda\cos\chi=g-g'w$$

and consequently

$$\cos\chi=\frac{g-g'w}{\sqrt{e-e'w^2}\sqrt{f-f'w^2}}$$

$$\sin\lambda=\frac{\sqrt{(e-e'w^2)(f-f'w^2)-(g-g'w)^2}}{\sqrt{(e-e'w^2)(f-f'w^2)}}$$

which gives

$$l\lambda\sin\chi=\sqrt{(e-e'w^2)(f-f'w^2)-(g-g'w)^2}$$

using this value in

$$C_{33}\frac{dw}{dt}=C_{11}l\lambda\sin\chi$$

and we obtain

$$t=\frac{C_{33}}{C_{11}}\int\frac{dw}{\sqrt{(e-e'w^2)(f-f'w^2)-(g-g'w)^2}}$$

The quantity under the radical in the denominator is a quartic function of w with real coefficients of the form

$$H+Kw+Lw^2+Mw^3+Nw^4$$

(but with $M=o$). The integral is therefore elliptic, and by the usual methods for transformation can be brought into the standard form. We also have

$$\chi=-rt+\left(\frac{C_{33}}{C_{11}}\right)^2 \int \frac{(g-g'w)wdw}{(e-e'w^2)\sqrt{(e-e'w^2)(f-f'w^2)-(g-g'w)^2}}$$

In the case of a sphere we will find by pursuing the same method as above

$$p=\text{const.}\quad q=\text{const.}\quad r=\text{const.}$$
$$u^2=v^2=w^2=\text{const.}$$

that is the velocity does not vary with the time, we will also find

$$t=\int \frac{dw}{(e-w^2)f-(g-rw)^2}$$

$$\psi=\int \frac{(g-dw)wdw}{(e-w^2)\sqrt{f(e-w^2)-(g-rw)^2}}-rt$$

We can assume now special cases of the motion, and attempt to find the integrals of our equations. If the motion in the direction of the axis of y is equal to zero we shall have

$$v=p=r=o$$

and our equations will become

$$C_{11}\frac{du}{dt}=-C_{33}wq$$

$$C_{33}\frac{dw}{dt}=C_{11}uq$$

$$C_{44}\frac{dq}{dt}=(C_{33}-C_{11})uw$$

Multiply the first of these by u, the second by w, and add

$$C_{11}u\frac{du}{dt}+C_{33}w\frac{dw}{dt}=-(C_{33}-C_{11})uwq$$

Combining this with the last, multiplied by q, and we obtain

$$C_{11}u^2+C_{33}w^2+C_{44}q^2=\text{const.}=F'$$

Multiplying the first by $C_{11}u$, and the second by $C_{33}w$, and adding and we have again

$$C_{11}^2u^2+C_{33}^2w_3=\text{const.}=E'$$

From this last equation we see that if we represent u by a function that increases with the time, we must represent w by one that decreases with the time. Suppose that we represent u by an elliptic function of the form $\sin\lambda t$, then we can represent w by one of the form $\cos\lambda t$, and then these conditions implied in the above relation between u and w will be satisfied. Now the equation

$$\frac{dw}{dt}=\frac{C_{11}}{C_{33}}uq$$

suggests the form of q. For we have

$$\frac{d\cos\lambda t}{dt}=-\lambda\sin\lambda t\,du\lambda t$$

but $u=\sin\lambda t$, then rq can be given as $du\lambda t$. It is obviously necessary to introduce other constants so we may write u,w and q as follows:

$$u=a_b\sin\omega t$$
$$w=a_c\cos\omega t$$
$$q=a_d\,du\omega t$$

There are here in all five constants

$$a_s, a_c, a_d, \omega, k$$

two of them are then arbitrary and may be taken as two of the constants of integration. Suppose now that the body suffers no displacement in the directions of the axes, that is the origin of x, y, z is fixed, and in consequence

$$u=v=w=o.$$

This gives for T

$$2T=C_{44}(p^2+q_e^2)+C_{66}r^2$$

Substituting again in our equations of motion and they become

$$\frac{dp}{dt}=-\frac{(C_{66}-C_{44})}{C_{44}}rq$$

$$\frac{dq}{dt}=\frac{(C_{66}-C_{44})}{C_{44}}pr$$

$$\frac{dr}{dt}=o \text{ or } r=\text{const.}$$

Represent the factor $r\,\frac{C_{66}-C_{44}}{C_{44}}$ by b_1 then these equations are

$$\frac{dp}{dt}=-b_1q$$

$$\frac{dq}{dt}=b_1p$$

from which follows

$$p^2+q^2=\text{const.}=a_1$$

We can then write

$$p=a_1\cos\omega t$$
$$q=a_1\sin\omega t$$

when ω is easily seen to be

$$=\left(\frac{C_{66}}{C_{44}}-1\right)r.$$

assume $p=q=o$. Then we have

$$2T=C_{11}(u^2+v^2)+C_{33}w^2+C_{66}r^2$$

and the equations of motion are

$$\frac{du}{dt}=vr$$
$$\frac{dv}{dt}=-ur$$
$$\frac{dw}{dt}=o \quad \text{or } w=\text{const.}$$
$$\frac{dr}{dt}=o \quad \text{or } r=\text{const.}$$

the first two gives

$$u^2+v^2=\text{const.}$$

or

$$u=b_1\cos\omega t$$
$$v=b_1\sin\omega t$$
$$w=g_1$$

these are the equations of a helix, that is the points of the body in this case describe helices whose common axis is parallel to the axis of z.

In concluding this chapter we wlll give a brief account of the integrals that we have denoted by A, B, C respectively. We had

$$A=abc\int_o^\infty \frac{du}{(a^2+u)\sqrt{(a^2+u)(b^2+u)(c^2+u)}}$$

assume $e^2=\frac{a^2-c^2}{c^2}$ and as usual $k^2=\frac{a^2-b^2}{a^2-c^2}$ with $k'^2=\frac{b^2-c^2}{a^2-c^2}$. Introduce a new variable x instead of u and defined by the equation

$$u=c^2\frac{c^2-x^2}{x^2}$$

From this we have

$$du=-c^2\frac{2e^2}{x^3}=-2\frac{(a^2-c^2)}{x^3}$$

and

$$a^2+u=\frac{(a^2-c^2)(x^2+1)}{x^2},$$

$$b^2+u=\frac{(b^2-c^2)x^2+(a^2-c^2)}{x^2},$$

$$c^2+u=\frac{a^2-c^2}{x^2},$$

and consequently

$$\sqrt{(a^2+u)(b^2+u)(c^2+u)}$$
$$=(a^2-c^2)\sqrt{(x^2+1)[(b^2-c^2)x^2+(a^2-c^2)]}$$

These being substituted in the value of A give us

$$A=-\frac{2abc}{a^2-c^2}$$

$$\int\frac{dx}{\frac{x^2+1}{x^2}\sqrt{(x^2+1)[(b^2-c^2)x^2+(a^2-c^2)]}}$$
$$=R\int\frac{dx}{\frac{x^2+1}{x^2}\sqrt{1+k'x^2}}$$

when

$$R=-\frac{2abc}{(a^2-c^2)^{\frac{3}{2}}}$$

Now let $x=\tan\theta$, the limits of θ being o and $\frac{\pi}{2}$ as those of x are manifestly o and ∞.

$$dx=\sec^2\theta d\theta$$

hence

$$A=R\int_o^{\frac{\pi}{2}}\frac{d\theta}{\frac{\sec\theta}{\tan^2\theta}\sqrt{1+k'^2\tan^2\theta}}$$

$$=\frac{R}{k^3}\int_o^{\frac{2}{\pi}}\frac{k\sin^2\theta d\theta}{\sqrt{1-k^2\sin^2\theta}}$$
$$=-\frac{R}{k^3}\left\{\int\frac{(1-k^2\sin^2\theta)d\theta-d\theta}{\Delta(\theta)}\right\}$$

and since $1-k^2\sin^2\theta=[\Delta(\theta)]^2$ this becomes

$$=-\frac{R}{k^4}\left\{\frac{E'-F'}{k}\right\}$$

and finally

$$A=-\frac{R}{k}\cdot\frac{dE'}{dk}$$

F' and E' denoting the complete elliptict integrals of the first and second kinds. We have next

$$B=abc\int\frac{du}{(b^2+u)\sqrt{a^2+u)(b^2+u)(c^2+u)}}$$

Making the same transformations as before this becomes

$$=R\int \frac{dx}{\frac{1+k'^2x^2}{x^3}\sqrt{1+k'^2x^2}}$$

$$=R\int \frac{\tan^2\theta\sec\theta\,d\theta}{(1+k'^2\tan^2\theta)^{\frac{3}{2}}}$$

$$=R\int \frac{\cos^3\theta\tan^2\theta\sec\theta\,d\theta}{[\Delta(\theta)]^3}$$

$$=R\int \frac{\sin^2\theta d\theta}{[\Delta(\theta)]^3}$$

$$=-\frac{R}{k^2}\left\{\int\frac{d\theta}{\Delta\theta}-\frac{d\theta}{[\Delta\theta]^3}\right\}$$

$$=-\frac{R}{k^2}\left\{F'-\frac{E'}{k'^2}\right\}$$

and finally

$$B=\frac{R}{k}\cdot\frac{dF}{dk}$$

Again we have for C the expression.

$$C=abc\int_0^\infty \frac{du}{(a^2+u)\sqrt{a^2+u)(b^2+u)(C^2+u)}}$$

Transforming as in the preceding cases this becomes

$$=R\int\frac{\tan\theta d\theta}{\Delta(\theta)}$$

$$=\frac{R}{k'^2}\left\{\int\frac{1+k'^2\tan^2\theta}{\Delta(\theta)}d\theta-\int\frac{d\theta}{\Delta\theta}\right\}$$

Integrating by parts this becomes

$$=\frac{R}{k'^2}\left\{\int\Delta\theta\tan-\int\frac{-k^2\sin^2\theta}{\Delta\theta}d\theta-F\right\}$$

which gives finally

$$C=\frac{R}{k'^2}[\Delta\theta\tan\theta-E]$$

The reader, desirous of further information upon the subject treated in this section, is referred to papers by Kirchhoff and Boltzman, in volumes 70, 71, 73, 78 of Crelles Journal, also one by Bjerkues, in the Proceedings of the Société des Sciences at Christiana for the year 1871. It is from these papers that the matter contained in the previous pages is drawn.

ON MAGNETIZATION OF STEEL DURING HARDENING.

From "The English Mechanic."

It is known that in production of permanent magnets, the steel is first hardened and then magnetized, because in hard steel, though the temporary magnetism is less, the fixation of the magnetism succeeds better. After Scoresby and E. Becquerel proved, however, that the tempory magnetism of steel continuously increases up to a dark red glow, it seemed probable, that remarkably powerful permanent magnets might be obtained if the magnetization were effected during the very process of hardening. Experiments in this direction were made by Ponitz in 1821, also by Robison, Aimé, and others. These researches, however, are not quite convincing in their results; partly because of the paucity of the experiments, and partly because of the lack of accurate description of the two methods used, and comparison of their results. Herr Holtz has therefore been led to take up the question anew, since, also, it is one of considerable theoretical and practical interest. His researches are described in the *Annalen der Physik.*

To get comparable results it was desirable to operate with two exactly similar steel bars, in the same way, with the same magnetizing force, except that in one case the force should be made to act after hardening, in the other, during this process. As means of magnetization, Herr Holtz used in the first half of his experiments an electro magnet, in the other a magnetizing spiral. The current was supplied by one to three Grove elements. The steel bars were round and of various thickness. They were all 125mm. long, and from the same works. The same bar could be used three times without any important difference appearing in its magnetic behavior. All the bars were quenched when at a bright red, so that during the magnetization all points had passed the dark red glow.

Each pair of bars was heated to the required degree as regularly as possible in a wood charcoal fire; then one bar was quenched, and afterwards the other, after and while the magnetizing force acted on it. The time of this action was fifteen seconds. Then the first bar was exposed to the same magnetizing force for the same time. The time of magnetization of both bars thus amounted to somewhat over thirty seconds, during which time the intensity of the current certainly remained constant. Other bars were being heated during the magnetizing process. Thus 20 to 30 pairs could be treated in 1¼ hour. The magnetism was measured by the method of oscillation, each bar, suspended by a long cocoon fibre, being let swing under the magnetic force of the earth, and observed with a telescope having cross threads.

We will not here pursue far the details of the two methods. Where an electro-magnet was used a portion of the end was left free of wire, and the coil was also protected from the water (used in the hardening process) by an envelope. The bars were fixed in holders, with which they were pressed against the bent poles of the electro-magnet. In the case of a magnetizing spiral being used, this was protected by being wound round a copper tube with end plates, and surrounded by a metallic cylinder. The width of the inner tube was more than double the diameter of the thickest bar to be inserted, so that the water might get freely to the latter; and the renewal of the water was insured by moving the case in it. Experiments were made, in both cases, with constant and with variable magnetizing forces, and under special conditions.

The general result was, that the magnetization during hardening is only conditionally superior to ordinary magnetization. The explanation of this follows from the explanation of the more special results, which Herr Holtz expresses in the following propositions:

The superiority of magnetization during hardening decreases,

1. Where the magnetizing force increases.

2. Where the thickness of the bar increases.

3. It is greatest, where, with weak magnetizing force, the bar is at the same time partly withdrawn from the action of the means of magnetization.

4. It is least, or passes into an inferiority, where with great magnetizing force a great thickness of bar is used.

The third proposition is only a consequence of the first, the fourth a consequence of the first and second. Herr Holtz proceeds to indicate what he considers the reason of these effects.

With regard to the practical bearing of the above, it appears that magnetization during hardening could furnish magnets which were over six times as strong as those produced by the ordinary method of magnetization. But their absolute strength was extremely small, because it was only with exceedingly weak magnetizing force that so great a difference appeared. The more the latter increased, the more did the difference diminish, till at length the difference became in favor of the ordinary mode of magnetization. With the maximum magnetizing force used, three large Grove elements and a coil of 600 windings, a bar with even the thickness of 6mm. was better magnetized after hardening than during it. But for production of strong permanent magnets stronger forces would probably be used.

Nor did the permanent magnetism of the bars magnetized during hardening seem to offer a greater resistance to external influences.

The conclusion therefore (from these and other considerations) is reached that magnetization during hardening presents no practical advantage whatever over the ordinary method of magnetization.

Boring for coal has commenced about half a mile on the Auckland side of the Mercer (New Zealand) station; there are encouraging signs of a good seam of coal underlying at no great depth from the surface. In 1878, 149,266 tons of coal were imported into the colony from Newcastle, N.S.W., and 138,984 tons were raised in New Zealand itself. The latter is supplied to consumers at about half the price of the former. Waikato coal is supplied to the railway authorities at 8s. 9d. per ton, and Bay of Island coals, one of the best steam coals known, to the steamers at 12s.

THE THEORY OF ICE MACHINES.

By M. LEDOUX, Mining Engineer.

Translated from "Annales des Mines" for VAN NOSTRAND'S MAGAZINE.

III.

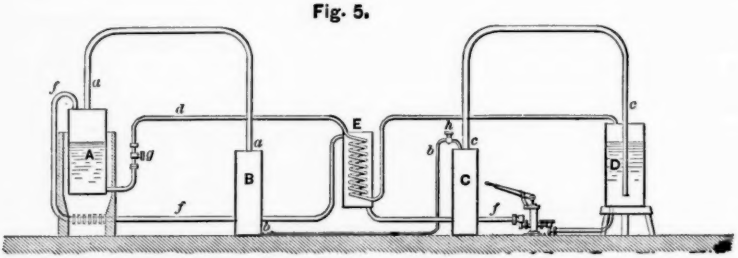
CHAPTER IV.

MACHINES EMPLOYING CHEMICAL ACTION.

§ 34. It remains to discuss the ice making machines which employ chemical affinity in their mode of action, and of which the ammonia machine of M. Carré is the type.

Fig. 5 exhibits the disposition of the parts of this apparatus. It consists of a boiler A which contains a concentrated solution of ammonia in water; this boiler is heated either directly by a fire as shown in the figure, or indirectly by pipes leading from a steam boiler. The condenser B communicates with the upper part of the boiler by the tube *aa*; it is cooled externally by a current of cold water. The *réfrigérant* C is so constructed as to utilize the cold produced; the upper part of it is in communication with the lower part of the condenser by means of the tube *bb*. The details of the construction are not shown in the figure. An absorption chamber D is filled with a weak solution of ammonia; the tube *cc* puts this chamber in communication with the refrigerant C.

Fig. 5.



The absorption chamber communicates with the boiler by two tubes. One *dd*, leads from the bottom of the boiler to the top of the chamber D; the other, *ff*, leads from the bottom of D to the top of the boiler. Upon the pipes *ff* is mounted a little pump whose use is to force the liquid from the absorption chamber where the pressure is maintained at about one atmosphere, into the boiler, where the pressure is from 8 to 12 atmospheres.

The change of temperature is managed through the attachments to the pipes *ff* and *dd* in a manner that will be easily comprehended by an inspection of the figure.

To work the apparatus the ammonia solution in the boiler is first heated. This releases the gas from the solution and the pressure rises. When it reaches the tension of the saturated gas at the temperature of the condenser, there is a liquefaction of the gas, and also of a small amount of steam. By means of the cock *h*, the flow of the liquefied gas into the refrigerant C is regulated. It is here vaporized by absorbing the heat from the substance placed here to be cooled. As fast as it is vaporized it is absorbed by the weak solution in D. The small quantity of watery vapor is carried along mechanically.

Under the influence of the heat in the boiler A, the solution is unequally saturated, the stronger solution being uppermost.

The weaker portion is conveyed by the pipe *dd* into the chamber D, the flow being regulated by the cock *g*, while the pump sends an equal quantity of strong solution from D back to the boiler.

While these exchanges are brought about in the solutions, there is also an exchange of temperatures whereby the weak liquid arrives cold in the absorption chamber, and the strong solution is delivered in the boiler hot.

The working of the apparatus depends upon the adjustment and regulation of the cocks g and h, and of the pump; by means of these, the pressure is varied, and consequently the temperature in the refrigerant C controlled.

It is seen that the working is similar to that of the machines described in the preceding chapters. The chamber D fills the office of aspirator, and the boiler A plays the part of compressor.

The mechanical force producing exhaustion, is here replaced by the affinity of water for ammonia gas; and the mechanical force required for compression is replaced by the heat which severs this affinity and sets the gas at liberty. We see then in advance that we shall again find a greater part of the equations already established in the discussion of the liquefiable gas machines.

§ 35. We will assume at first, that under the influence of the heat applied to the boiler, ammonia gas only is driven off, and no steam. We will assume a certain weight of the gas to enter the boiler in a state of solution; being heated, it will be separated from the water, requiring a certain quantity of heat which we will call Q′. Then, being conducted to the condenser, it will be cooled and then liquefied, and will impart to the water surrounding the coils a quantity of heat Q_1. In the refrigerant it is evaporated, borrowing from the exterior a quantity of heat Q; it is next absorbed by the liquid in the chamber D, disengaging a certain amount of heat to the liquid (which may be deducted from the total amount required in the boiler); and, finally, it is reconveyed to the boiler, where it arrives in its original condition. By reason of the exchange of temperature effected at E, all the heat of the weak solution going out of the boiler, is restored to the strong solution entering it, so that the changes of temperature in the water are effected without expenditure of heat.

In the complete cycle if we neglect the small amount of work performed by the pump, and the heating and cooling due to contact with the air, it is clear that all the heat from external sources, being Q′ from the boiler, and Q from the refrigerant, will be equal to the amount Q_1 carried away by the water of the condenser.

We have then

$$Q' = Q_1 - Q$$ and the

efficiency will be expressed by

$$\frac{Q}{Q_1 - Q}$$ which is identical

with that found for the machines depending on mechanical action.

Q′ the quantity of heat which it is necessary to expend in order to produce the quantity Q of negative calories, being equal to $Q_1 - Q$, has the same value as the quantity AW_r, the calorific equivalent of the mechanical work expended in the machines previously discussed, to produce this same quantity Q of negative calories. We proceed to show that between the same limits of temperature in the condenser and refrigerant, and for the same value of Q, the quantity Q′ in this class of machines, is equal, very approximately at least, to the quantity AW_r.

We arrive then at this remarkable result; that in all the ice machines, when they work between the same limits of temperature, the theoretic quantity of negative heat produced is exactly same for each calorie expended, whether it is directly produced by chemical action, or indirectly under the form of mechanical work.

But as a calorie represented by 424 kilogrammeters costs in the best heat motors an expenditure of at least 10 calories in the fire, it would seem that the chemical machines possess a considerable advantage over all the others, since in these latter the heat is employed directly, and not under the expensive form of mechanical work. Practically, however, this advantage is much less than that which seems to result from the above calculations; as we will proceed to show.

§ 36. We will assume the hypothesis mentioned in the beginning of the preceding section, and determine the quantities Q′, Q_1 and Q in terms of the temperatures, the pressures and weights of the gas employed.

We will preserve the notations of the

previous chapter. T_1 being the absolute temperature of the gas as it enters the condenser; T_1' its absolute temperature in the condenser, and T_2 the absolute temperature in the refrigerant.

Let m be the weight of the gas considered, occupying the volume V_2 at the temperature T_2, and under the pressure P_2 at its entrance into the absorption chamber.

Let AU be the internal heat at the temperature T; qe the heat necessary to raise a kilogram of water from 0° to 1°.

After the gas has been absorbed by the water, the absolute temperature of the mixture will be T'_2.

During the process of absorption of the gas, there is an amount of external work accomplished equal to $P_2(V_2-w)$, w being the volume of water.

The difference in internal heat before and after this operation is equal to this external work. We have then

$$q_e{}'_2+mAU_2'-q_{e_2}-mAU_2=AP_2(V_2-w).$$

The solution is conveyed to the boiler, and there heated until all the gas is driven off. It then occupies the volume V_1 under the pressure P_1, and at the temperature T_1.

The necessary quantity of heat Q″ is equal to the difference in quantities of internal heat, augmented by the exterior work accomplished. This work is equal to $P_1(V_1-w)$ less the work of the pump, $(P_1-P_2)w$.

We have then

$$Q''=q_{e_1}-q_e{}'_2+mAU_1-mAU_2 \\ +AP_1(V_1-w)-A(P_1-P_2)w.$$

Adding this equation to the preceding, member to member, we find

$$Q''=q_{e_1}-q_{e_2}+mA(U_1-U_2)+AP_1V_1-AP_2V_2$$

This equation is established without taking account of the effect of exchange temperature. There is furnished to the solution which enters the boiler a quantity of heat precisely equal to $q_{e_1}-q_{e_2}$. The quantity of heat Q′ to be supplied by the boiler, in order to bring the pressure of the gas from P_2 to P_1, and from the temperature T_2 to T_1 is then

$$Q'=mA(U_1-U_2)+AP_1V_1-AP_2V_2 \quad (107)$$

The equations 101 and 105 gave, in case of compression by a mechanical force,

$$AW_r=mA(U_1-U_2)+AP_1V_1-AP_2V_2$$

which is identical with the preceding.

We have then $Q'=AW_r$ provided that the temperature T_1 in the case where the change of pressure of the gas is obtained by the heat combined with the chemical action, is the same as in the case where the change is due to a mechanical force. Experiment proves that it is nearly so.

It appears that the temperature to which it is necessary to heat the ammonia solution to obtain a given pressure is higher as the solution becomes weak. Now in the ice machines the solution conveyed to the boiler contains rather less of the gas as the pressure in the refrigerant becomes more feeble. We understand therefore how the temperature T_1 ought to increase as the temperature T_2 of the refrigerant diminishes. Unfortunately, precise experiments upon this point are wanting.

A series of observations made by M. Rouart upon a Carré machine is herewith given.

The first column of each table gives the absolute pressures in atmospheres and kilograms; the second the temperatures observed in the boiler; the fourth, the temperatures of water in the condenser; the fifth column gives the temperatures of the liquefied gas corresponding to the pressures in the first column (see table in § 22); the temperatures are those of the interior of the condenser, and are naturally more elevated than the exterior.

In the case of mechanical compression the final temperature T_1 is related to the initial temperature and to the initial and final pressures as expressed by the equation (100)

$$T_1=T_2\left(\frac{P_1}{P_2}\right)^{\frac{AB}{Cp}}$$

The third column of the table gives the temperatures calculated by this formula, supposing $T_2=243$ and $P_2=11{,}918$.

For the mean pressures the calculated temperatures coincide nearly with the observations. For the higher pressures the calculated pressures are higher than the observed. But it is necessary to remark that in this case the watery vapor mixed with the gas exerts a greater influence, and that the true gas press-

Pressure in Boiler.		Temperature of Boiler		Temperature of water of condenser.	Temperature inside of condenser.	Difference.	Remarks.
Atm.	Kilog.	Observed.	Calculated				
		Degrees.	Degrees.	Degrees.	Degrees.	Degrees.	
1½	15,501	48	—	9	—	—	
2	20,668	58	—	9	—	—	
2½	25,835	65	—	9	—	—	
3	31,002	70	—	9	—	—	
3½	36,169	75	—	9	—	—	
4	41,336	80	—	9	—	—	
4½	46,503	84	—	9	—	—	
5	51,670	88	—	9	—	—	
5½	56,837	92	—	9	—	—	
6	62,004	94	—	9	—	—	
6½	67,171	100	—	10	—	—	The gas liquefies and the apparatus begins to work.
7¼	74,921	106	106	10	15.0	5.0	
7½	77,505	108	109	10	16.1	6.1	
8	82,672	112	116	12	18.0	6.0	
8½	87,839	116	121	13	20.0	7.0	
9	93,006	120	127	14	21.7	7.7	
9½	98,173	124	132	15	23.3	8.8	
10	103,340	128	137	17	25.1	8.1	
10½	108,507	132	142	17	26.7	9.7	
11	113,674	136	147	19	28.1	9.1	
12	123,998	142	156	24	31.0	7.1	
13	134,332	146	164	30	32.8	2.8	
14	144,666	152	172	35	36.0	1.0	
15	155,000	156	180	37	38.0	1.0	
15	155,000	158	180	39	38.0	—	

Absolute pressures.		Temperature of Boiler		Temperature of condenser water (observed).	Temperature of interior of condenser (calculat'd)	Difference of temperatures.	Observations.
Atm.	Kilog.	Observed.	Calculated				
		Degrees.	Degrees.	Degrees.	Degrees.	Degrees.	
3	31,002	73	—	8	—	—	
4½	46,503	90	—	8	—	—	
5	51,670	94	—	8	—	—	
5½	56,837	100	—	8	—	—	
6	62,004	103	—	8	—	—	
6½	67,171	106	—	8	—	—	The liquefied gas appears.
7	72,338	110	—	9	14.1	—	
7½	77,505	118	109	10	16.1	6.1	
8	82,672	124	116	12	18.0	6.0	
8½	87,839	130	121	14.5	20.0	5.5	
9	93,006	136	127	15	21.7	6.7	
9½	98,173	140	132	16	23.3	7.3	
10	103,340	146	137	17	25.1	8.1	
10¾	111,089	147	145	19	27.4	8.4	
11¾	121,423	148	153	16 (?)	30.4	14.4 (?)	
13	134,332	148	164	16 (?)	32.8	16.8 (?)	
14	144,666	154	172	35	36.0	1	
15	155,000	160	180	38	38.0	0	
15½	160,167	163	180	38	40.0	1	

ures ought to be sensibly less than the pressures which have served as a basis for calculation.

The condensation in the condenser and the evaporation in the refrigerant, are brought about exactly as in the case of the machines acting by mechanical force. We shall have then, as in § 27,

$$Q_1 = mc_p(T_1 - T_1') + mr_1',$$

$$x_2 r_2 = q_1' - q_2,$$

$$Q=m(1-x_2)r_2=mr_2-m(q_1'-q_2),$$

$$m=\frac{V_2}{u_2+\frac{0.001}{\delta}}=V_2 v_2$$

$$Q'=Q_1-Q.$$

The two following tables give the results of calculations for one cubic meter of ammonia gas, for temperatures in the condenser ranging from +15° to +40°. In the first the temperature of the interior of the refrigerant is taken at −15°. In the second table it is −30°.

The numbers in the last column are calculated on the supposition that a kilogram of coal burned yields 4000 calories.

First case: $t_2=-15°$, $m=1^k.932$.

Temp. interior of condenser.	Temperature of boiler.	Calories removed by condenser.	Negative calories produced.	Calories expended.	Theoretic performance.	Theoretic result per kilogram of coal.
deg.	deg.	cal.	cal.	cal.	cal.	cal.
15	67.77	638.71	564.83	73.88	7,645	30,580
20	81.74	640.76	554.49	86.27	6,427	25,708
25	95.69	642.61	543.98	98.63	5,515	22,060
30	109.61	644.12	533.29	110.83	4,813	19,252
35	123.47	645.53	522.43	123.10	4,244	16,976
40	137.27	646.57	511.31	135.26	3,779	15,116

Second case: $t_2=-30°$, $m=1^k.023$.

Temp. interior of condenser.	Temperature of boiler.	Calories removed by condenser.	Negative calories produced.	Calories expended.	Theoretic performance.	Theoretic result per kilogram of coal.
deg.	deg.	cal.	cal.	cal.	cal.	cal.
15	106.07	361.77	293.04	68.73	4,263	17,052
20	121.61	364.09	287.57	76.52	3,771	15,084
25	137.13	366.31	282.01	84.30	3,345	13,380
30	152.61	368.38	276.35	92.03	3,003	12,012
35	168.03	370.31	270.60	99.71	2,714	10,856
40	183.38	372.09	264.71	107.38	2,465	9,860

The results indicated by the preceding tables are large; they vary from 9,860 to 30,580 negative calories for each kilogram of coal burned. We are far from attaining such results in practice.

We have omitted in our calculations to take into account two conditions which modify largely the theoretical results:

1st. The necessity of cooling the absorption chamber so that the solution of the gas may be readily accomplished.

2d. The influence of the water carried along with the gas.

We will now examine the influence of these two causes of loss.

§ 37. When ammonia gas dissolves in water, considerable heat is disengaged.

M. M. Fabre and Silbermann have measured this heat of solution, and found it equal to $514.^{cal}3$ for each kilogram of gas dissolved.

The liquid of the absorption chamber being employed continually in dissolving the gas from the refrigerant, rises rapidly in temperature, and as the solubility diminishes with the temperature, it soon reaches a condition at which it ceases to work. To insure successful working it is necessary, therefore, to treat the absorption chamber to a current of cold water in such a manner as to maintain a constant temperature. We will suppose this to be the same as that of the condenser t_1'.

If we denote by Q_1' the quantity of heat, of which the absorption chamber is relieved, we shall evidently have

$$Q_1+Q_1'=Q+Q'$$

or

$$Q'=Q_1+Q_1'-Q.$$

On the other hand, the gas arriving at the condenser, is always mixed with a certain quantity of steam, usually about 6 or 8 per cent. By employing a solution of calcium chloride instead of pure water for a solvent, the amount of watery vapor is reduced to about three per cent.

The presence of the steam reduces the efficiency to a notable extent. It carries off a portion of the heat of the boiler, and, having arrived in the refrigerant, it does not evaporate, but, by holding a portion of the ammonia, prevents it from volatilizing. It impedes the action then, nearly in the same way as the waste spaces in the mechanical action machines, but to a greater extent.

We will proceed to determine the influence of this introduction of water.

Let m, as before, be the weight of gas sent out from the boiler; μ the weight of water accompanying it, and the quantities r and q affected by the index e, shall relate to the water.

When the mixture passes into the condenser, the steam becomes liquid, and absorbs a certain weight m' of gas; and we have

$$m'=\frac{0.001\beta_1'\mu}{v_1} \qquad (108)$$

β_1' being the coefficient of solubility by volume of the gas in water whose temperature is t_1'; $\frac{1}{v'}$ being the weight of a cubic meter of gas at this temperature.

According to Carius, the coefficient of solubility of ammonia, a gas in water, is represented by the empirical formula

$$\beta=1049.624-29.4963t+0.676873t_2 \\ -0.0095621t^3.$$

The quantity of heat Q_1 which will be absorbed by the condenser, is equal to the quantity of heat necessary to lower the temperature of the weight m of gas from t_1 to t_1', plus the quantity of heat necessary to liquefy the weight $m-m'$ of gas, plus the quantity of heat necessary to liquefy, and raise to the temperature t_1' the weight μ of steam, plus the heat disengaged by the solution of the weight m' of gas.

We shall have then

$$Q_1=mc_p(T_1-T_1')+(m-m')r_1'+\mu \\ (q_{e_1}-q_{e_1}'+r_{e_1})+m's_1', \qquad (109)$$

calling s_1' the heat disengaged by the solution of one kilogram of ammonia gas in water having the temperature t_1'.

The mixture passing into the refrigerant, a certain quantity of the liquefied gas is volatilized until the pressure and temperature become equal, respectively, to P_2 and T_2, the pressure and temperature of the refrigerant. The water will retain in solution a weight m'' of gas, given by the equation,

$$m''=\frac{0.001\beta_2\mu}{v_2} \qquad (110)$$

The quantity of gas volatilized $(m-m')x_2$ is found by the equation

$$\left.\begin{array}{l}(m-m')x_2r_2+(m'-m'') \\ \qquad s_2=(m-m')(q_1-q_2)+ \\ +\mu(q_{e_1}'-q_{e_2})+m'c_p(T_1'-T_2)\end{array}\right\} \qquad (111)$$

The quantity of negative head realized is

$$Q=(m-m')(1-x_2)r_2 \qquad (112)$$

or

$$\left.\begin{array}{l}Q=(m-m')(r_2-q_1'+q_2)(m'-m'')-s_2 \\ \qquad -\mu(q_{e_1}'-q_{e_2})-m'c_p'(T_1'-T_2)\end{array}\right\} (113)$$

The quantity of heat Q_1' which it is necessary to supply to the absorption chamber in order to maintain a constant temperature, is equal to the heat arising from the solution of $m-m''$ weight of gas, minus the heat necessary to raise the weight m of gas and the weight μ of water, from T_2 to T_1'.

$$Q_1'=(m-m'')s_1'-mc_p(T_1'-T_2) \\ -\mu(q_{e_1}'-q_{e_2}) \qquad (114)$$

The quantity of heat Q' which it is necessary to employ at the boiler, is equal to $Q_1+Q_1'-Q$. We have then, applying the above values,

$$Q'=m[c_p(T_1-T_1')+s_1']+(m-m') \\ [\lambda_1'-\lambda_2-c_p(T_1'-T_2)]+ \\ +(m'-m'')(s_1'-s_2)+\mu(\lambda_{1^e}-q_{e_1}')$$

The heat of solution s varies probably with the temperature and the pressure of the gas, but we do not know the law of this variation, and we, therefore, assume this quantity to be constant and equal to 514.3 calories, as found by Favre and Silbermann, for ordinary temperatures and pressures.

Making $s_1'=s_2=s$ in the above equation it becomes

$$Q'=m[c_p(T_1-T_1')+s]+(m-m') \\ [\lambda_1'-\lambda_2-c_p(T_1'-T_2)]+\mu(\lambda^e{}_1-q_{e_1}) \qquad (115)$$

we have further

$$m=\frac{V_2}{u_2+\frac{0.001}{\delta}}=\frac{V_2}{v_2}$$

§ 38. The two following tables exhibit results; the two cases of § 36 are taken, supposing that the weight of watery vapor carried over is 5 per cent. of the weight of the gas circulating.

If we compare the figures of these tables with those of § 36, we find that the cooling of the absorption chamber, and the presence of watery vapor, reduce the efficiency to a considerable extent.

We see further that the useful effect diminishes in proportion as the temperature is lowered in the refrigerant, but that the results remain the same for the same temperature of the condenser.

First case: $t=-15°$, $m=1^k.932$, $m'=0^k.3091$, $\mu=0^k.0966$.

Temperature of interior of condenser t_1'.	Pressure in condenser P_1.	Temperature in boiler t_1.	Heat carried away from condenser & absorption chamber Q_1+Q_1'.	Disposable negative heat Q.	Heat expended Q'.	Ratio of efficie'cy theoretic $\frac{Q}{Q'}$	Efficie'y per kilogram of coal $4{,}000\frac{Q}{Q'}$	Weight of gas dissolved by water carried over m'.
Degrees.	Kilog.	Degrees.	Cal.	Cal.	Cal.	Cal.	Cal.	Kilog.
15	74,504	67.77	1,584.43	485.69	1,098.74	0.442	1,768	0.400
20	87,925	81.74	1,583.92	477.47	1,106.45	0.431	1,724	0.407
25	103,073	95.69	1,584.80	470.63	1,114.17	0.422	1,688	0.421
30	120,083	109.61	1,583.59	461.79	1,121.80	0.411	1,644	0.426
35	139,054	123.47	1,577.94	448.70	1,129.24	0.397	1,590	0.412
40	160,112	137.27	1,564.13	427.78	1,136.35	0.376	1,506	0.364

Second case; $t_2=-30°$, $m=1^k.023$, $m''=0^k.147$, $\mu=0^k.0511$.

Temperature of interior of condenser t_1.	Temperature in boiler t_1.	Heat carried away from condenser & absorption chamber Q_1+Q_1'.	Disposable negative heat Q.	Heat expended Q'.	Ratio of efficiency theoretic $\frac{Q}{Q'}$	Efficiency per kilogram of coal $4{,}000\frac{Q}{Q'}$.	Weight of gas dissolved by water carried over m'.
Degrees.	Degrees.	Cal.	Cal.	Cal.	Cal.	Cal.	Kilog.
15	106.07	859.46	258.59	600.87	0.430	1,721	0.212
20	121.61	859.88	254.08	605.80	0.419	1,676	0.215
25	137.13	861.21	250.57	610.64	0.410	1,641	0.223
30	152.61	861.38	245.75	615.63	0.399	1,596	0.225
35	168.03	857.67	238.92	618.75	0.386	1,544	0.218
40	183.38	852.93	227.99	624.94	0.365	1,460	0.193

In the machines employing mechanical power, the efficiency on the other hand diminishes with the temperature of the refrigerant.

§ 39. In the practical manufacture of artificial ice, we estimate the performance at about 1200 or 1500 negative calories for each kilogram of coal burned, which is about 80 per cent. of the above figures. The difference here between theory and practice may fairly be attributed to external losses of temperature, to imperfect action in the exchanges of heat, and to expenditure of work in driving the pumps.

The constructors of sulphur dioxide machines claim a practical result of 2500 calories per horse power per hour. As a good engine consumes two kilograms of coal per horse power per hour, we are afforded a means of comparing the two kinds of apparatus in the matter of economy, and the result is in favor of the chemical action machines. The latter also afford the advantage of low temperatures.

In the sulphur dioxide machine, a lower temperature than $-12°$ is not attained without loss of useful effect, while in the ammonia machine $-25°$ and $-30°$ are readily and economically obtained.

We will not enter here upon questions of a purely practical character which affect the comparative values of the several ice machines, as our object has been simply to establish the theoretic conditions under which they work.

Appendix.

Note upon the determination of the latent heat of vaporization, also of the specific heat of sulphur dioxide and ammonia in the form of liquid.

It was shown in section 27 that the relation between the pressure, specific volume and temperature of a liquefiable gas, being represented by the equation

$$Pv=BT-CP^n, \qquad (116)$$

the constants B, C and n can be determined by means of the coefficient of

dilatation, and the experiments of Regnault upon the compressibility of gases.

These constants are

	For Sulphur Dioxide.	For Ammonia.
B	13.882	52.4943
C	3.8455	43.7144
n	0.44487	0.32685

Regnault determined also the elastic forces of these substances at different temperatures, and established the empirical formula

$$\log F = a + ba^t + c\beta^t.$$

This form not being convenient for calculation, we have preferred to take the formula called Roche's

$$P = \alpha a^{\frac{t}{1+mt}} \qquad (117)$$

and we have calculated the three constants a, α and m for both sulphur dioxide and ammonia.

These constants are

	For Sulphur Dioxide.	For Ammonia.
	$a=15840$.....	43474.64
log.	$a=4.1991752$.....	4.6382260
	$\alpha=1.04135$.....	1.0386605
log.	$\alpha=0.0176387$.....	0.0164736
	$m=0.0043129$.....	0.0040112

Finally M. Regnault found for the specific heat of sulphur dioxide 0.15438, and of ammonia gas 0.50836.

On the other hand Clausius established between the latent heat r, the absolute temperature T, the pressure P, and the quantity u the relation

$$\frac{r}{u} = AT\frac{dP}{dt}$$

or

$$\frac{r}{APu} = \frac{T\frac{dP}{dt}}{P} \qquad (118)$$

u is the increase of volume of a unit of weight of a volatile liquid when transformed into vapor.

If v is the specific volume of the vapor, we have

$$u = v - \frac{0.001}{\delta},$$

δ being the density of the liquid, and consequently

$$APu = ABT - ACP^n\frac{0.001.AP}{\delta} \qquad (119)$$

The constants B, C and n being known, the equation will give APu.

Knowing APu we find r by eq. 118.

$$r = APu\,\frac{T}{P}\frac{dP}{dt}$$

or in consequence of eq. 117

$$r = APu.\frac{T.la}{(1+mt)^2} \qquad (120)$$

The equation 120 will give r in terms of T and APu.

Finally it was shown in § 27 that the quantity λ, that is, the total heat of vaporization satisfies the equation

$$\lambda = \lambda_0 + c_p t - \frac{AC}{n}(P^n - P_0^n)$$

At temperature zero we have

$$\lambda_0 = r_0$$

then it becomes

$$\lambda = r_0 + c_p t - \frac{AC}{n}(P^n - P_0^n) \qquad (121)$$

an equation in which P_0 represents the pressure of the vapor at zero, c_p the specific heat of the vapor at constant pressure, and r_0 the latent heat at zero.

The heat of the liquid

$$q = \lambda - r.$$

We shall have then

$$q = r_0 + c_p t - \frac{AC}{n}(P^n - P_0^n) - APu\frac{Tla}{(1+mt)} \qquad (122)$$

and the specific heat of the liquid

$$c = \frac{dq}{dt}$$

The equations (119), (120), (121) and (122), involving laborious calculations, we can replace the second member by empirical expressions of the form $A' + B't + C't^2$, and then calculate the constants by means of three values taken at the two extremities and middle of the thermometric scale, and previously determined by aid of these equations.

We thus find for

Sulphur Dioxide

$$APu = 8{,}243 + 0{,}0196t - 0{,}000116t^2$$
$$r = 91{,}396 - 0{,}2361t - 0{,}000135t^2$$
$$\lambda = 91{,}396 + 0{,}12723t - 0{,}000131t^2$$
$$q = 0{,}36333t + 0{,}00004t^2$$
$$c = 0{,}36333 + 0{,}00008t.$$

For Ammonia

$$APu = 30{,}154 + 0{,}08861t - 0{,}000059t^2$$
$$r = 313{,}63 - 0{,}6250t - 0{,}002111t^2$$
$$\lambda = 313{,}63 + 0{,}3808t - 0{,}000282t^2$$
$$q = 1{,}0058t + 0{,}001829t^2$$
$$c = 1{,}0058 + 0{,}003658t.$$

The specific heat of liquid ammonia is nearly equal to that of water. This result, though astonishing at first, is comprehended when we reflect that the specific heat of the gas at constant pressure 0.50836 is higher than that of steam (0.4805). It would be interesting to verify by experiment the theoretical conclusion.

The results obtained here for ammonia are, however, only approximate, for we need, in order to determine the constants of eq. (116), the coefficient of dilatation of this gas, and at present it is not known.

To facilitate calculations upon the ice machines, we have prepared the following tables for sulphur dioxide and ammonia. They give for each 5° the heat of the liquid q, the total heat of vaporization λ, the latent heat of vaporization r, the internal latent heat ρ, the external latent heat $\mathrm{AP}u$, and the weight of a cubic meter of vapor $\frac{1}{v}$, for the temperatures between −40° and +40°.

TABLE I.—SULPHUR DIOXIDE (SATURATED).

Temperature centigrade. t	Absolute temperature. T	Pressure in kilo. per meter. P (Regnault)	Total heat. λ	Heat of liquid. q	Heat of vaporization. r	Latent heat external. $\mathrm{AP}u$	Latent heat internal. ρ	u	Weight of vapor per cubic meter. $\frac{1}{v}$
—30	243	3,908	87,461	—10.864	98,325	7,551	90,774	0.822	1,215
—25	348	5,082	88,134	— 9,058	97,192	7,681	89,511	0.641	1,558
—20	253	6,519	88,799	— 7,251	96,050	7,805	88,245	0.507	1,967
—15	258	8,265	89,458	— 5,441	94,899	7,923	86,976	0.406	2,456
—10	263	10,366	90,111	— 3,629	93,740	8,035	85,705	0.328	3,038
— 5	268	12,874	90,757	— 1,814	92,571	8,142	84,429	0.268	3,719
0	273	15,840	91,376	0,000	91,396	8.243	83,153	0.221	4,518
5	278	19,322	92,029	1,818	90,211	8,338	81,873	0.183	5,447
10	283	23,378	92,655	3.637	89,018	8,427	80,591	0.153	6,514
15	288	28,074	93,275	5,459	87,816	8,511	79,305	0.129	7,740
20	293	33,474	93,888	7,283	86,605	8,589	78,016	0.109	9,132
25	298	39,645	94,495	9,108	85,387	8,661	76,726	0.093	10,718
30	303	46,659	95,095	10,936	84,159	8,727	75,432	0.079	12,500
35	308	54,585	95,689	12,765	82,924	8,787	74,137	0.068	14,513
40	—	63,496	96,276	14,597	81,679	8,841	72,838	0.057	16,750

TABLE II.—AMMONIA GAS SATURATED.

Temperature centigrade. t	Absolute temperature. T	Pressure in kilo. per meter. P (Regnault)	Total heat. λ	Heat of liquid. q	Heat of vaporization. r	Latent heat external. $\mathrm{AP}u$	Latent heat internal. ρ	u	Weight of vapor per cubic meter. $\frac{1}{v}$
—40	233	7,187	297,967	—37,268	335,235	26,515	308,720	1.564	0.639
—35	238	9,302	299,955	—32,963	332,918	26,980	305,938	1.230	0.812
—30	243	11,918	301 952	—28,528	330,480	27,438	303,042	0.976	1.023
—25	248	15,120	303,934	—23,002	327,936	27,902	300,034	0.782	1.276
—20	253	19,003	305,901	—19,384	325,285	28,858	296,927	0 633	1.577
—15	258	23,669	307,855	—14,676	322,531	28,812	293,719	0.516	1.932
—10	263	29,225	309,794	— 9,875	319,669	29,262	290,407	0.424	2.348
— 5	268	35,797	311,719	— 4,983	316,702	29,708	286,994	0.352	2.840
0	273	43,475	313,630	0,000	313,630	30,154	283,476	0.294	3.385
5	278	52,405	315,527	5,075	310,452	30,594	279,858	0.247	4.020
10	283	62.707	317,418	10,241	307,177	31,034	276,143	0.210	4.737
15	288	74,504	319,279	15,498	303,781	31,470	272,311	0.179	5.543
20	293	87,925	321,133	20,848	300,285	31,902	268,383	0.154	6.447
25	298	103,073	322,979	26,288	296,691	32,332	264,359	0.133	7.446
30	303	120,083	324,800	31,820	292,980	32,754	269,226	0.116	8.547
35	308	139,054	326,611	37,443	289,168	33,182	255,986	0.101	9.766
40	313	160,112	328,431	43,196	285,235	33,604	251,631	0.089	11.074

A NEW GRAPHICAL METHOD FOR FINDING THE CENTER OF GRAVITY OF A POLYGON.

By J. WOODBRIDGE DAVIS, C.E.

Written for Van Nostrand's Magazine.

A paragraph in the article on *A New Center of Gravity Formula* in June No. of this Magazine, suggested to the writer a graphical method for determining the centers of gravity of polygons, which seems to be new and much better than existing methods. The course of argument by which it was reached is, however, too long to be used as the foundation of its truth, since a very short demonstration upon more familiar principles serves the same purpose. Of the former a brief outline only will, therefore, be given.

The polygon is supposed to be generated by a varying, broken line, which moves—

(*a*) from one vertex, as origin;

(*b*) along the two sides and all the diagonals which radiate from the vertex, as directrices;

(*c*) and parallel to the remaining sides of polygon, as a director;

(*d*) until it reach and coincide with them.

This vertex may be called the *principal vertex*, the two adjacent sides, the *principal sides*, the diagonals which radiate from the vertex, the *principal diagonals;* and the other sides may be called the *remaining* or *generating sides.* [It will be found convenient to retain this nomenclature, although this conception of the problem shall be discarded.]

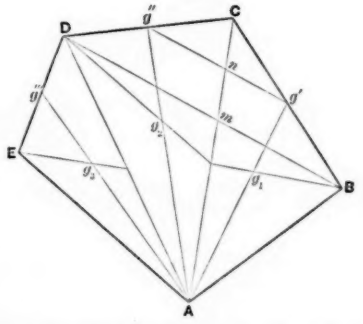
It is evident that the c. of g. [center of gravity] of generatrix describes a straight line. Also, that, if x denote variable distance of the c. of g. from principal vertex, along the line of gravity, and y represent the corresponding magnitude of generatrix,

$$y = ax.$$

In consequence, formula (1), of article in June No., applies. For this case it reduces to $\frac{2}{3}$D, D being the distance from origin to last position of c. of g. Therefore, if this last position be found, and joined with principal vertex by a straight line, and the latter be trisected, the point of trisection further from vertex is common c. of g. of the generatrix when in every position at once.

But this is not c. of g. of polygon, because it has been found with regard to positions and comparative lengths only of the generating sides, whereas the spaces they severally describe depend also upon their inclinations to the directions of their progress. So, we must not find actual c. of g. of generatrix in last position, but the c. of g. of space described in final instant; this we must connect, as before, with principal vertex, and trisect. The space described in last instant is a series of rectangles, having the generating sides for bases, but with altitudes depending upon their several inclinations to the directions of advance. To find comparative values of these rectangles, leads to the method of this paper, which can be more easily demonstrated as follows:

Fig. 1.



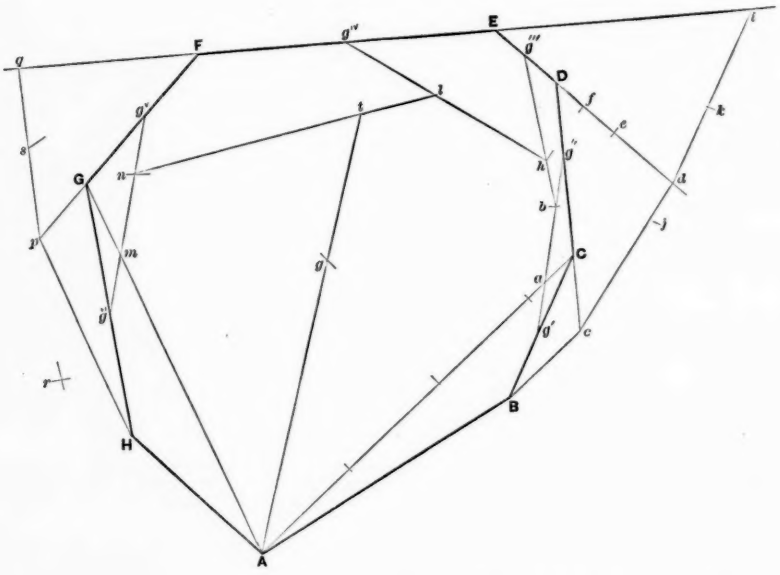
Suppose A,B,C, etc., to be any polygon. [Consult Fig. 1, which, though a pentagon, serves to illustrate the general statements of the text.] Let A be the principal vertex. Draw the principal diagonals. The polygon is now divided into triangles. The points g_1, g_2, g_3, etc., are found to be the centers of gravity of these. The c. of g. of first and second

triangles is on line joining g_1 and g_2, and divides this line into segments reciprocally proportional with the areas of the triangles. Of first three triangles the c. of g. is on line joining point last found and g_3, and its position may be found according to the same law. The c. of g. of polygon can finally be obtained by this method of procedure.

But, suppose a number of lines, equal to the number of triangles, to diverge from the principal vertex, each passing through one of the points g_1, g_2, etc. Establish on each of these a point thrice as far from vertex as half the distance to corresponding c. of g. of triangle. If, now, with these new points g', g'', g''', etc., instead of g_1, g_2, etc., we proceed in exactly same manner to find a common c. of g., on supposition that g', g'', etc., are centers of gravity of the triangles respectively, the point so found will, evidently, be in each direction three-halves the distance from principal vertex that the true c. of g. of polygon is. Therefore, if the false c. of g. be connected with principal vertex by a straight line, this will contain the true c. of g. of polygon, and the latter will be at two-thirds its length from vertex.

This is the first step of the new method. Its advantages are: The points g', g'', etc., are much more easily found than g_1, g_2, etc., since we know they are the middle points of the generating sides. Thus, for the labor of determining the centers of gravity of the triangles, is substituted the far less work of bisecting the same number of lines. Again, the construction is on a little larger scale, and is not crowded about the spot where the c. of g. of polygon will be found.

Fig. 2.



GENERAL METHOD.

To demonstrate the general method of finding c. of g. of g', g'', etc., let us consider the octagon of Fig. 2. A is chosen as principal vertex. AB, AH, are, accordingly, the principal sides. Bisect remaining sides. Draw $g'g''$. Intersect it by principal diagonal to vertex between. Lay off $g'a$ in opposite direction from g''. Thus is found b, which is c. of g. of g', g''. That this is so appears from Fig. 1, where

$$ABC : ACD :: Bm : mD :: g'n : ng'',$$

and from the fact that the c. of g. of g', g'', should divide $g'g''$ into segments inversely proportional with areas repre-

sented by these points. Join b, g'''. The c. of g. of these divides, likewise, the distance between inversely as area of first two triangles is to area of third. To find this point, draw Bc, ‖ 1st diagonal, AC, until it meet the 2d generating side at c, and draw cd ‖ 2d diagonal AD. [The diagonals themselves need not be drawn.] The areas in question are as dD to DE, or, if dD be bisected at e, as eD is to Dg'''. Reciprocate these segments by laying off g'''D from e to f; then, ‖ eb, draw fh. h is c. of g. of g', g'', g'''. Again join h,g^{IV}, and draw di ‖ 3rd diagonal AE. hg^{IV} must be divided inversely as iF is divided at E. iE might be bisected as was dD, and the same process used as before. A method generally simpler is to lay off hg^{IV} from i along id to j, draw, ‖ F j, the line Ek, or enough of it to mark intersection, and lay off ik,—which is proportional with area represented by h,—*in opposite direction from the other end*, g^{IV}. Thus is found l.

So might we continue round the polygon to the last side GH, describing for each successive side a line ‖ next diagonal yet unused till it intersect the side prolonged, and thus obtaining a ratio of length of side to length of prolongation, which would enable us to find the common c. of g. of that side, and all the preceding sides. The operation is exactly the same, and as simple for one side as for any other. To use the method purely, we should find b from the ratio of cC to CD; but in this instance it is found always preferable to obtain the segments directly on the line of gravity itself.

In order to occupy less space outside the polygon for construction lines, apply the method in both directions around polygon, commencing at each side of principal vertex, and continuing each way to meet the result of the other application. Thus, as before, n is found by reciprocation of the segments made by diagonal AG. Join n and l. To find where on this line is c. of g. of all the generating sides—as it may be called, although it is not indeed the c. of g. of these, but the c. of g. of space described in last instant by symmetrical motion of these sides from principal vertex—draw Hp ‖ AG, and pq ‖ AF. Now, as qF : Fi, so should lt be to tn. Then t is c. of g. of generating sides. nl was laid off on qp from q to r, and ‖ ir was drawn Fs. qs was laid off from l to t.

Join t with principal vertex. Then is g, the point of trisection of tA further from vertex, the c. of g. of the octagon.

qi is the *area line*. If its length be denoted by l, and by p, its perpendicular distance from A, then

$$\text{area of polygon} = \tfrac{1}{2}\, pl. \qquad (1)$$

A description of the only other general method of finding graphically the center of gravity of a polygon, is quoted from Professor Du Bois' *Graphical Statics*, p. 29, in order that the two may be compared. It is as follows:

"Generally, we divide up the given area into triangles, trapezoids, rectangles, etc., and reduce the area of each of these figures to a rectangle of assumed base. The heights of these reduced rectangles will then be proportional to the areas, and hence to the force of gravity acting upon them; *i.e.*, to their *weights*. Consider then these heights as forces acting at the centers of gravity of the partial areas. Construct the *force polygon* by laying them off one after the other. Choose a pole and draw lines from it to the beginning and end of each force. These lines will give the sides of the *funicular* or *equilibrium polygon*. Anywhere in the plane of the figure, draw a line parallel to the first of these pole lines (S_0). Produce it to intersection with the first force (P_1), prolonged if necessary. From this intersection draw a parallel to the second pole line (S_1), and produce to intersection with second force (P_2). So on to the last pole line, which produce to intersection with first pole line. Through this point the resultant must pass, and of course it must be parallel to the forces.

"Now suppose the parallel forces all revolved say 90°, the points of application remaining the same. Evidently the new force polygon will be at right angles to the first, as also the new pole lines, each to each. It is unnecessary then to form the new force polygon. The directions of the new pole lines are given by the old, and this is all that is needed.

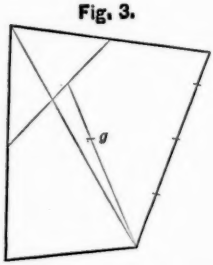
"Anywhere, then, in the plane of the figure, draw a line (S_0') perpendicular to the first pole line (S_0) previously drawn, and prolong to intersection with new

direction of first force (P_1'). Through this point draw a perpendicular (S_1') to second pole line, to intersection with new direction of second force (P_2') and so on. We thus find a point for new resultant, parallel to new force direction. Prolong this resultant to intersection with first, and the center of gravity is determined."

A few words, which cover merely the preliminary part of the operation, will, notwithstanding this, pointedly enforce the advantage of the method of this paper. If n be the number of sides of the general polygon, where, by the new method, it is requisite simply to bisect $n-2$ sides, it is necessary by the old,—to carry the solution thus far,—to actually construct $n-1$ parts, as trapezoids, triangles and rectangles, and determine their centers of gravity.

We have very frequently need to determine the c. of g. of the quadrilateral. The process is very simple. Let Fig. 3 illustrate. Bisect two adjacent sides, and join the points of bisec-

Fig. 3.

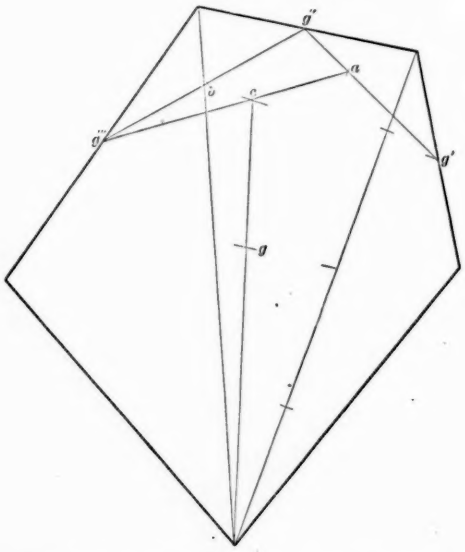


tion. Intersect the line by the diagonal from opposite vertex. Reciprocate the segments of the line joining points of bisection, and, finally, join point last found with opposite vertex, and determine its point of trisection farther from vertex. The last is the c. of g. of quadrilateral.

METHODS BY INTERSECTIONS.

When a polygon possesses few sides, we may lessen the work by employing *methods of intersection.* Let Fig. 4

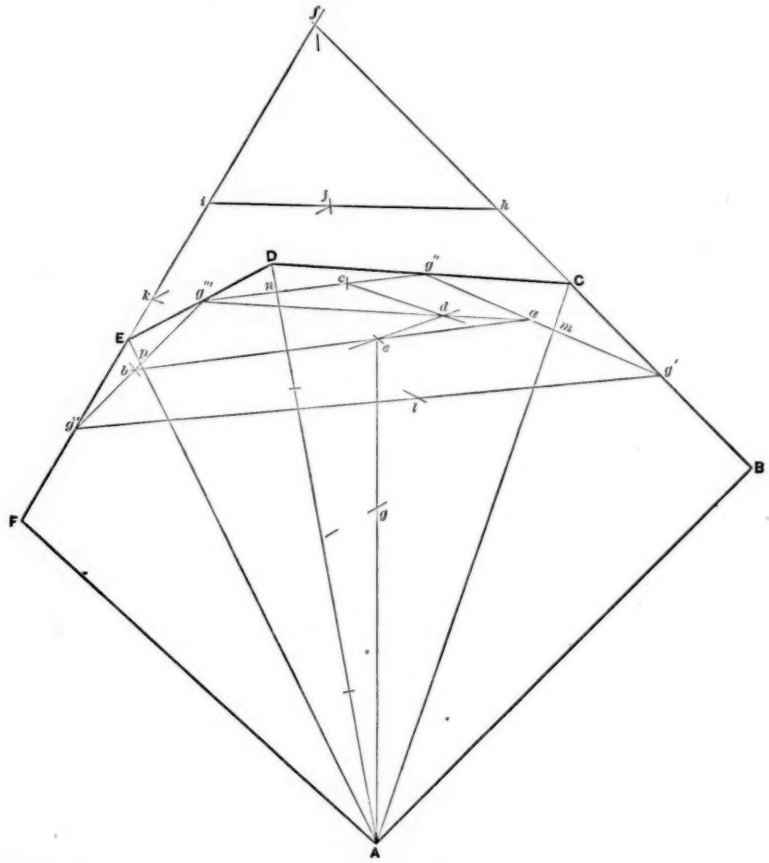
Fig. 4.



exemplify. Join g',g'', and reciprocate segments, to obtain a. ag''' contains c. of g. of the three sides. Instead of comparing areas of triangles, draw $g''g'''$, and find point b. [This is very close to diagonal, since this nearly bisects $g''g'''$.] bg' also contains c. of g. So join point of intersection c, with principal vertex, and trisect. g is the c. of g. of pentagon.

Fig. 5 is a hexagon. Draw $g'g''$ and

Fig. 5.



$g'''g^{iv}$, and reciprocate segments. Now ab contains c. of g. of sides. Next, find c. of g. of first three sides according to method described in last paragraph. That is: join a with g''' and intersect this line by cg'. d is c. of g. of three sides. Therefore dg^{iv} contains c. of g. of the four sides; and the point of intersection, e, is the c. of g. Hence, g is c. of g. of polygon.

Another method by which, at greater expense, a sharper intersection can be made, is also displayed in Fig. 5. As before, ab is one line of gravity. Intersect this by line joining the c. of g. of sides CD, DE with c. of g. of BC, EF. c is former point. To find latter, produce BC, FE, till they meet, if possible. Lay off fh, fi, respectively equal to BC, EF. Intersect hi by Af. Then, as hj is to ji, so let $g^{iv}l$ be to lg'. We can find l conveniently by drawing $jk \parallel hg^{iv}$, and $kl \parallel ig'$ e, the intersection of ab and cl, is c. of g. of sides, as before found; and g is c. of g. of polygon.

A third method of determining the c. of g. of the polygon by intersection is as follows: [again see Fig. 5.] Find the line ab as before; then describe $\parallel$ thereto, and at two-thirds its distance from vertex, another line. The latter contains c. of g. of polygon. Now change the pole or principal vertex to some other vertex, and proceed in same manner.

We shall thus obtain two lines of gravity, whose intersection is the c. of g. of polygon.

These methods by intersections can be applied to heptagons, octagons, etc., but the general rule, as illustrated in Fig. 2, will be found more convenient generally for polygons the number of whose sides exceeds six.

SHORT APPROXIMATE METHOD.

Let us next examine an exceedingly easy, but merely approximate method, which may prove of value in practical applications where great accuracy is not required. It might be called the *method by reciprocation of segments*, as the c. of g. on each line of gravity is approximately found simply by reciprocation of the segments made by intersecting diagonal.

Suppose it were desirable to ascertain as easily as possible the approximate position of c. of g. of Fig. 2. Draw $g'g''$; reciprocate segments to find b; draw bg'''; reciprocate segments made by next diagonal AD, to find h nearly. Again, invert segments of hg^{iv}, made by AE, to find l. So might we continue; but, as the error is apt to accumulate in same direction, it is far better to apply the method from both sides. The two errors then tend to cancel each other, and we gain, moreover, the certainty of having one additional point correctly found. This is n. Draw nl. Reciprocate segments made by last remaining diagonal, AF. From approximate position of t, a still closer position of g is derived.

This operation is very rapid. The result it produces for Fig. 2 will be found very nearly right. It would be better, though, if the method should be applied to three sides one way and three the other, instead of to four one way and two the other. It is not a safe method to use when polygon is very irregular or contains re-entrant angles: It may be depended upon whenever polygon is well balanced. For a regular hexagon it has been found quite accurate. Who uses this method must discover from more extensive experience than the writer has had for what classes of shapes it can most be relied upon.

For the quadrilateral, Fig. 3, it is, of course, correct. For the pentagon, Fig. 4, find a, and reciprocate segments of ag''', made by 2d diagonal, to find c. It is in this case also very close. By this method are found for Fig. 5 the points a, b. Simply reciprocate segments of line joining them, made by middle diagonal, AD. Thus is e found with great accuracy, and thence g.

SECOND GENERAL METHOD.

There is another exact general method which is worthy of mention for the reason that nearly all the work is thereby performed within the polygon. But it has the disadvantage of being longer than the other general method of this paper. Consult Fig. 5 again. Find a as before. This operation is indeed a part of this method, and was borrowed to be the first step of the other general method. Connect a with middle point of third side, g'''. Divide this into segments inversely, proportional with area of first two triangles, and area of third as follows:

$$\text{ADC} : \text{ACB} :: g''m : mg'.$$

Also,

$$\text{AED} : \text{ADC} :: g'''n : ng''.$$

In order to convert $g'''n$ to the scale of $g'g''$, project $g'''n$ upon $g'g''$ by lines $\parallel nm$. The distance so found lay off from g'' on $g'g''$ prolonged. Next, lay off $a'g'''$ from g' on $g'f$, and divide it proportionally with the lengths on the area line, $g'g''$. Thus is found d. Draw dg^{iv}. Then, for fourth triangle, convert $g^{iv}p$ into scale of $g'g''$ by projecting it upon $g'''g''$ by lines parallel with pn, and this upon $g'g''$ by lines $\parallel$ with nm. So proceed till all the triangles have been considered. $g'g''$ with its prolongation is the area line. On it are lengths of bases, $g'm$, mg'', etc., of rectangles, equivalent in order with the triangles, the constant height being the distance between A and C measured normally to $g'g''$.

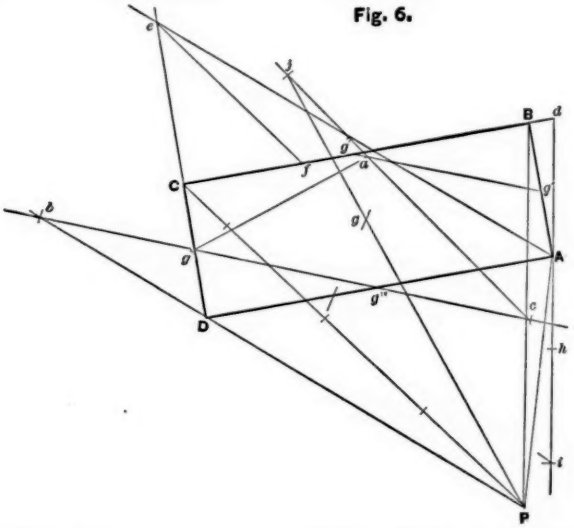
The disadvantage of this method is that the labor increases not as the number of sides of polygon, but more rapidly. The labor may be greatly reduced by application of method from both ends, which procedure seems preferable for all the methods. The advantage of the method is, as before remarked, that the construction lines are almost entirely within polygon. The writer has applied the method with success to a very irregular octagon with re-entrant angles. But, as it seems to be far behind the other method, further space shall be to it denied.

POLES.

Instead of one vertex of a polygon, any point in the plane can be employed as a pole. From it draw lines to the vertices, and proceed according to any foregoing rule. The demonstration of the truth of these methods covers the general problem, of which all previous examples are but special cases. When pole is outside of polygon, the latter is composed of a series of positive and negative triangles, the algebraic sum being the area thereof. If the pole be situated upon one side, or its prolongation, evidently, one triangle vanishes. If pole be placed at intersection of two sides, whether prolonged or not, two triangles vanish. Such a position is, for instance, the point *f* of Fig. 5, or any one of the vertices.

The generality of this method enables us to construct very interesting theorems respecting the position of c. of g. of a polygon referred to any pole. The only restriction as to situation of pole is that it must not be at an infinite distance. With the pole thus far removed, the problem becomes indeterminate.

Fig. 6.



EXAMPLES.

To find c. of g. of rectangle represented in Fig. 6, we have chosen the pole P. Connect this with the four vertices. Bisect the four sides. Commence with any one as AB, and follow round the polygon to last DA, using any method already explained, or proceed in opposite directions from first and last sides. Indeed, if we choose, we may consider the polygon PABCDAP. As this is a hexagon, submit it to any treatment described in connection with Fig. 5. Now P may be considered principal vertex, and PA, AP, may be considered principal sides. This conception of the problem is not, however, necessary.

Reciprocate segments of $g'g''$, made by pole line to included vertex. So *a* is found. In like manner reciprocate segments of $g^{iv}g'''$, made by pole line PD to vertex between. That is, *lay off* $g'''b$ *in opposite direction from other end*, g^{iv}. *c* is thus determined. *c* represents c. of g. of two triangles PDC, PDA, as *a* corresponds to c. of g. of triangles PCB, PBA. Connect *c*, *a*. *ca* is a line of gravity of polygon's sides with respect to the pole P. It corresponds with line *ab* of Fig. 5.

Now, if the general method illustrated in Fig. 2 be employed, A must be projected along a line ∥ PB, upon CB prolonged. C*d* represents area of first two triangles. To represent the other two on same scale, project A, along a line ∥ PD, upon DC prolonged. A*e* corresponds to H*p* of Fig. 2. Next draw *ef* ∥ PC.

fd, as *qi* of Fig. 2, represents total area. Therefore, lay off *ac* from *d* to *h*, as *nl* in Fig. 2 is laid off from *q* to *r*. ‖ *fh* draw C*i*, as F*s* in Fig. 2 is drawn ‖ *ir*. *di* corresponds to the two first triangles whose c. of g. is represented by *a*. Consequently, *di* must be laid off from other end, *c*, toward *a*. Accordingly, *i* falls at *j*. This is c. of g. of the sides for the pole P. Draw *j*P. Trisect this. The point of trisection, *g*, further from P is c. of g. of rectangle.

After finding the gravity line *ac*, we might obtain in same manner the c. of g. of $g'g^{IV}$, and of g'', g''', and connect the two points. This would furnish a second gravity line. Its intersection with first would be at *j*. Thence find *g* as before. This exactly corresponds with second method used for Fig. 5.

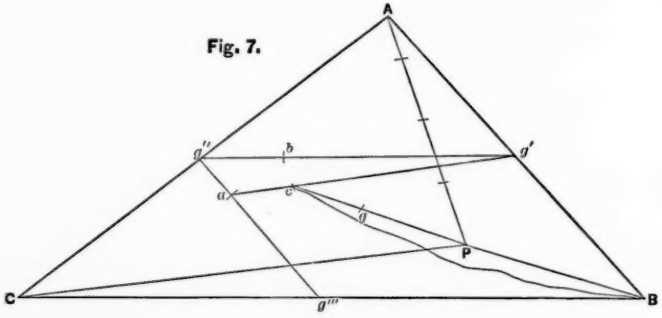


Fig. 7.

Fig. 7 is a triangle. Choose any pole as P, connect this with three vertices. Bisect three sides. Join any two points of bisection, and reciprocate the segments made by pole line to included vertex. Join point so found with third point of bisection. Thus is found *ag'*. Join two other points of bisection, reciprocate segments, and join point so found with remaining point of bisection, exactly as before. So is found *bg'''*. Connect the point of intersection, *c*, of *ag'*, *bg'''*, with pole, and trisect. The point of trisection, *g*, farther from pole, is c. of g. of triangle. This mode of procedure is exactly the same as that employed for Fig. 4. We may consider PBACBP a pentagon if we choose.

The last two examples are designed to illustrate the generality of the methods of this paper. It will often be found convenient to place the pole at the intersection of the two sides when prolonged, instead of at a vertex. The work is no greater so, and the component triangles may be, sometimes, made less cramped than would occur if any vertex should be used as a pole. Such an example is presented in Fig. 11.

POSITIVE AND NEGATIVE AREAS.

To further test the generality of these methods, let us find the c. of g. of the quadrilateral A B C D, Fig. 8, whose boundary crosses itself. As explained in connection with Fig. 4, p. 293, April No. of this Magazine, one branch of this figure must be considered positive, and the other negative. Proceeding in same manner as with Fig. 3, bisect two adjacent sides, BC, CD, connect the points of bisection by a straight line, $g'g''$, and reciprocate segments of this made by line AC, from opposite vertex to included vertex. This determines point *a*. Draw *a*A and trisect. *g* is the c. of g. of the quadrilateral. To ascertain position of c. of g. on supposition that the figure is all positive or negative, it must be considered a hexagon A*m*CD*m*BA. Fix the pole at *m*, as this reduces four triangles to nothing. Now, by second method used for Fig. 5, abbreviated on account of the four zero triangles, join points of bisection $g'g'''$. $g'g'''$ is a line of gravity for the sides, referred to the pole *m*. To find c. of g. of sides, prolong BA, DC; from point of intersection lay off these two sides, and join the extremities by line *cd*. Divide this by *mb*. Divide $g'g'''$ in same ratio. Thus $g'''h = ef$. Join *hm*; trisect. *g* is c. of g. of A*m*CD*m*BA.

The polygon with positive and nega-

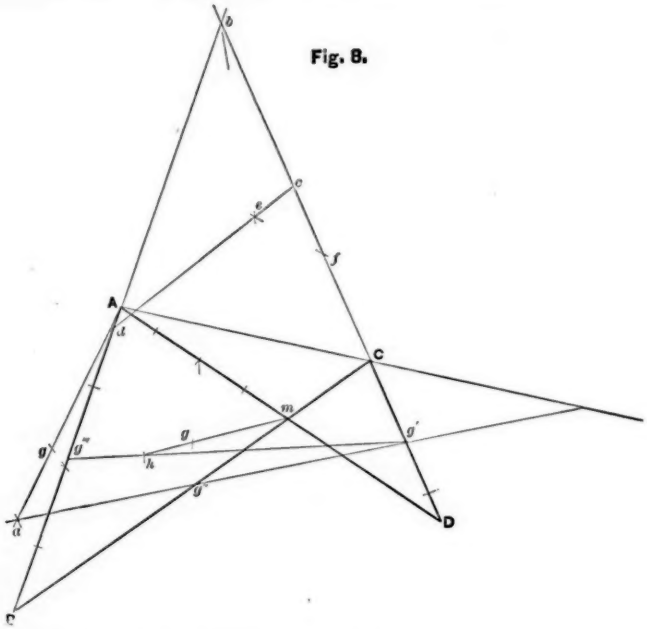


Fig. 8.

tive portions, which has been reverted to several times in previous articles of the writer's, is met with in practice. A practical example requiring the algebraic summation of its area, was presented in the article on *mensuration* in April No. of this Magazine. Touching more nearly the subject of present paper, we have, in discussions of strains in girders and trusses, to determine centers of gravity of *load* and *moment areas*, which may under some—and usual—conditions consist of parts having opposite signs.

AREAS OF DIFFERENT INTENSITIES.

Another interesting and practical case is that of the polygon possessing parts of different space intensities, or, to speak mechanically, portions filled with materials of different specific gravities. If a polygon possess two such parts, and the line of separation be straight, assume the pole at one extremity thereof, and apply the general method both ways from the pole, as in Fig. 2, till the constructions meet at the other extremity. Here we shall have the area line qi [Fig. 2], qF representing the area of one part, and Fi that of the other. At the same time we shall have gravity line nl. The only departure from the general method, then, is that nl must not be divided proportionally with areas, that is, in same ratio with qF, Fi, but in the ratio

$$\overline{q\mathrm{F}}.\ \gamma' : \overline{\mathrm{F}i}.\ \gamma'', \qquad (2)$$

where γ', γ'' are the specific gravities. In this problem one specific gravity may be expressed in terms of the other; so we may substitute for (2) the ratio

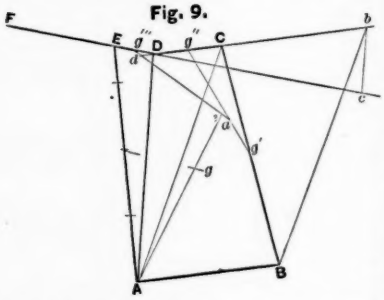
$$\overline{q\mathrm{F}} : \overline{\mathrm{F}i}.\ \gamma \qquad (3)$$

Therefore, lay off on Fi a length equal to $\overline{\mathrm{F}i}\ \gamma$, and use latter instead of former in the completion of the general construction.

If there be three parts, and two straight separatrices, locate, if possible, the pole at their intersection. But, in any case, whatever be shape of separating lines, the whole may be considered as one polygon, in manner indicated for Fig. 5 of article in June No. of this Magazine; and the parts of area line representing the various portions of polygon may at the last be multiplied by their several proper factors.

Fig. 9 illustrates a practical example

Fig. 9.



of this kind. The figure is copied from Fig. 15 of Professor Eddy's article on *Graphical Statics*, page 5, vol. XVII of this Magazine. ABCD is a retaining wall. FD is the surface of earth sustained. AE is a vertical line separating the triangle of earth AED from the rest. The wall and this portion of earth are supposed to act as a buttress against the remaining material AEF. Consequently, the c. of g. of ABCDEA must be determined. The weight of a cubic foot of wall is assumed to be double that of same bulk of earth.

Let A be pole. Bisect, BC, CD, DE. join g', g'', and reciprocate to find a. Draw ag'''. Parallel to AC, AD, successively, draw Bb, bc. Draw Dd ‖ ca. Lay off $ae=g'''d$. Join e,A. g is c. of g. required.

The same construction, line for line, serves for the solution of similar problems involved in discussion of Fig. 16 of Professor Eddy's article, where the wall has a backward cant, and the triangle of earth must be considered negative.

Fig. 10.

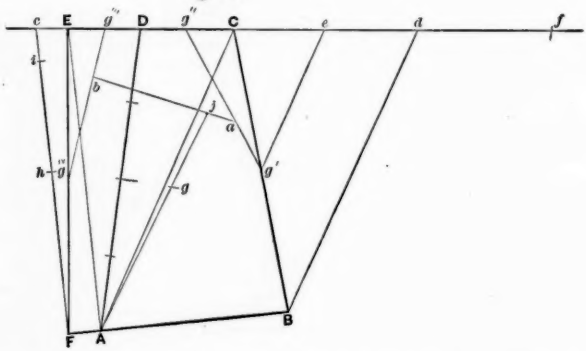


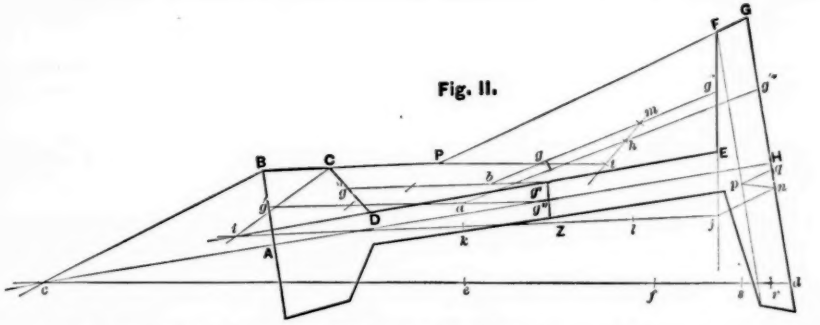
Fig. 10 represents the case where the dividing line, AD, passes through no vertex. The polygon is, therefore, ABCDE FA. Such is the case of a retaining wall ABCD, with a loose rock backing ADEF. Assume the specific gravity of solid wall to be three halves that of backing. Bisect BC, CD, DE, EF. Apply method both ways. So is found gravity line ab. Draw, ‖ AE, AC, respectively, Fc, Bd. The areas are as cD: Dd. But, draw $g'e$, also ‖ AC, and lay off $g''e$ from d to f. Lay off ab from c to h. Draw Di ‖ fh, and lay off $aj=ci$. Thence we find g to be c. of g. of combined mass.

Behind the backing of stone we might still have to regard a triangle of earth. Suppose, for present purpose, that FcE is this triangle. Then, we must consider the polygon ABCDEFecFA. Space shall not be given to illustration of this case.

GIRDER CROSS-SECTION.

Fig. 11 is the cross-section of a girder. The problem is, to find the position of its neutral fiber. Consider the polygon ABCDEFGH. Prolong BC, GF till they intersect. Let this point be the pole. Regard, now, separately, the pentagons PBAHGP and PCDEFP. Accordingly, bisect BA, AH, HG and CD, DE, EF. Treating both at same time, draw $g'g''$, $g^{vi}g^{v}$, and reciprocate segments of both, made by PA, PD, respectively. Draw ag''', bg^{iv}. To find center of gravity of larger pentagon, draw Bc ‖ to PA, cd ‖ PH; lay off $de=ag'''$, draw Hf ‖ Ge, and lay off $g'''h=df$. h is c. of g. of *generating sides* of pentagon, with re-

Fig. 11.



spect to P. Similarly, draw Ci ∥ PD, ij ∥ PE; lay off $jk=bg^{iv}$, draw El ∥ Fk, and lay off $g^{iv}m=jl$, mh is a line of gravity of all the sides of figure with respect to P, and contains the c. of g. of same sides with respect to same pole. To discover this point, determine the area ratio between the triangles PdG and PjF equal each in area with corresponding pentagon. Convert PjF into equivalent PnF by drawing jn ∥ PF. Draw pole-line np.

The following theorem will here be found useful, and, perhaps, hereafter. Let it be called

RULE E.

The area of any triangle is equal to one-half the rectangle of distance between any two vertices measured in any direction, and the distance from third vertex to opposite side measured normally to the other distance.

[*Note.*—In the signification of the word *side* in above theorem, is included the infinite extension of that line.]

Hence, area PnF=$\frac{1}{2}p$F×perpendicular distance from P to HG. Therefore, draw pq ∥ PF. Gq represents area of small pentagon, and dG represents area of large one. Because Gq is laid off backward upon dG, we know that the two areas have opposite signs, and qd is their algebraic sum. As always before, lay off mh from d to r. Draw Gs ∥ qr, just as Fs in Fig. 2 is drawn ∥ ir. ds pertains to larger area. Hence it must be laid off from m, which corresponds to small area, toward h. The extremity falls at t. Join t,P, and trisect by means of three equal distances, laid off consecutively from P toward F. g is c. of g. of polygon composing one-half cross section of girder. Through g draw ∥ GH the line Z through girder. Z occupies position of neutral fibre.

It will be noticed that to Fig. 11 we have really applied the general method, first illustrated in Fig. 2. That is the nonagon PBAHGFEDCP has been considered. P is the principal vertex. PB, CP, are the two *principal* sides. Commencing with BA, we have applied the method around one way to the side HG, which has been selected as the area line. Next, commencing at the other end, CD, we have proceeded so far as EF, which, for that portion of polygon, is the area line. To convert latter scale into former, we must, according to the rule, project j along a line ∥ to next pole line, PF, until it intersect FG. This line is infinitely long. From this position a line must be drawn ∥ to next pole line, PG, till it intersect GH. This is equivalent to drawing jn, as has been done. But, since the bases, PF, PG, of the two triangles are different, np, pq, must be drawn.

The area of girder cross-section equals rectangle of dq and perpendicular distance thereto from P, as is expressed by equation (1).

If the intersection of BC, FG do not fall in a convenient place, choose one of the vertices as pole, for instance, H or A. With H as pole, treat the pentagons HABCDH, HGFEDH, join the centers of gravity of their generating sides, and divide this line into segments inversely proportional with their areas. The process is exactly the same as that just described.

Fig. 11 may be divided into a series of five trapezoids. If AB, GH were not normal to AH, it would contain five trapezoids and two triangles; yet the

process would be exactly the same in length. In common with the numerical rules for determining centers of gravity and moments of inertia of polygons in general, and girder cross sections in particular, described in the June and July numbers of this Magazine, the present graphical method possesses the advantage, over all hitherto proposed, that it remains as brief for a complex arrangement of the perimeter as for a simple one. For this very reason there are a few simple arrangements to which the common method is more advantageously applied. For example, if CD, EF, were perpendicular to AH, and DE were ‖ thereto, we should have the common double T girder composed of three rectangles, and should find the method by force and funicular polygons more concise.

The ordinates of B, C, etc., G, may be made any equimultiples of their present lengths, AH remaining constant, if it be desirable to have the drawing on a larger vertical scale.

A figure with symmetrical axis is favorable to the old method in so much as the axis bisects the parallel bases of all the trapezoids, and saves a certain amount of labor which would otherwise be necessary in the determination of centers of gravity of the separate parts. The difference in labor is, therefore, greater for such a shape as ABCDEFGH, or any general polygon. It must be remembered, too, that the result of present method is the determination of position of *center of gravity* (so often required, though not in present case), while the old method, for this problem, need only be carried so far as the determination of a *gravity line*.

To determine one co-ordinate of c. of g. of an irregular figure, mensurated by breadths taken at intervals, lay off these breadths from a base line at their proper distances apart, and apply method as above.

DIVISION INTO TRAPEZOIDS,

If the breadths of such a figure be taken at equal distances apart, dividing the space into a series of equally long trapezoids, describe a system of lines parallel to the breadths and each midway between two. These new lines represent approximately the lines of gravity of the trapezoids; and the portions included within the figure represent exactly the proportionate intensities of gravity in the trapezoids. Find the common c. of g. of all, by means of the force and funicular polygons.

Thus are the loads upon trusses and arches commonly divided; and this is the method proposed as an approximate one by the writers who treat such problems. The error is not apt to be great in any practical case of a long series of trapezoids; but, whatever it be, it can be readily evaluated as follows, with aid of the formula

$$\frac{D^2(K-A)}{12V} \qquad (4)$$

which is formula (28) of *Center of Gravity* article, published in June No. of this Magazine. D is the distance between each two consecutive breadths, K is length of last, and A, the length of first, breadth. V is total area.

Fig. 12.

Let the vertical line of Fig. 12 represent the force polygon already used. The ends of the pole lines are shown; but the pole is not. Let F=length of force line.

Then $$V=FD, \qquad (5)$$

and the correction formula reduces to

$$\frac{\frac{1}{12}D(K-A)}{F}. \qquad (6)$$

On the last pole line lay off $bc=K-A$. Also lay off $bd=\frac{1}{12}D$. Then, ‖ ac, draw de. be is the amount of error in position of c. of g. of series as before found. If K – A be positive, move approximate

c. of g. a distance *be* away from A. If A>K, move c. of g. this distance toward A. When A=K, the approximate method is correct.

This correction, whether made numerically or graphically, is important, because it converts, with a trifling amount of work, a *good approximate* method into a perfectly accurate one.

SERIES OF SOLIDS.

Exactly the same method may be applied to a series of solids, when each is represented by some form of the quadratic equation. The resultant of gravity for each solid may be assumed to act through its mid-section. The volumes being already calculated, we have at once a system of parallel forces given in intensity and position. The resulting position of line of gravity of series may be graphically found and corrected as in case of trapezoids, and the new position will be absolutely true.

There appears to be no other method for a solid, divided as above into parts by a system of equi-distant, parallel planes. The seeming uncertainty of assuming that the weight of each portion acts through its mid-section, has prevented writers from employing such an approximate method as they have for the series of trapezoids. The present writer has, in a work on *Earthwork Calculation*, applied the graphical method to a series of railroad solids of unequal lengths. The engineer who encounters such problems will, however, find the numerical rules enunciated in the June No. of this Magazine shorter than even this graphical method. The latter is useful as a check upon the other.

The writer has refrained from showing the special constructions for more than a few practical examples, because the object of this article is to exhibit a new fundamental method, for which ample application can be found throughout the field of Graphical Statics.

LINKAGES.

A KINEMATIC DISCUSSION OF THE DIFFERENT FORMS OF ARTICULATED LINKS WITH ESPECIAL REFERENCE TO PEAUCELLIER'S CELL.

By J. D. C. DE ROOS.

Translated from "Revue Universelle des Mines" for VAN NOSTRAND'S MAGAZINE.

III.

Somewhat more complicated again is the combination required for describing a right line making a given angle with the line through the fixed points.

This arrangement is shown in Fig. 23, and is composed of two of the transformed cells (*systeme varié*) and four ordinary Peaucellier elements, all of the same power C^2 with the exception of the positive elements LL′ and NN′, whose powers are equal respectively to C'^2 and C''^2.

We have in this case

$$AM=\frac{C^2}{OA},\quad AP^2=AM^2-C^2=\frac{C^2(C^2-OA^2)}{OA^2}$$

$$AF=\frac{C^2}{AP}=\frac{C.OA}{\sqrt{C^2-OA^2}},$$

$$AH^2=C^2+AF^2=\frac{C^4}{C^2-OA^2}$$

$$AK=\frac{C'^2}{AH}=\frac{C'^2}{C^2}\sqrt{C^2-OA^2}$$

$$OK=OA-AK-OA-\frac{C'^2}{C^2}\sqrt{C^2-OA^2}$$

$$OB=\frac{C''^2}{OK}=\frac{C''^2}{OA-\frac{C'^2}{C^2}\sqrt{C^2-OA^2}}$$

If, furthermore, A describes a circle which passes through O, we have

$$OA=2r\cos\varphi$$

and we take $C^2=4r^2$ and

$\frac{C'^2}{C^2}=\tan\alpha$ we obtain

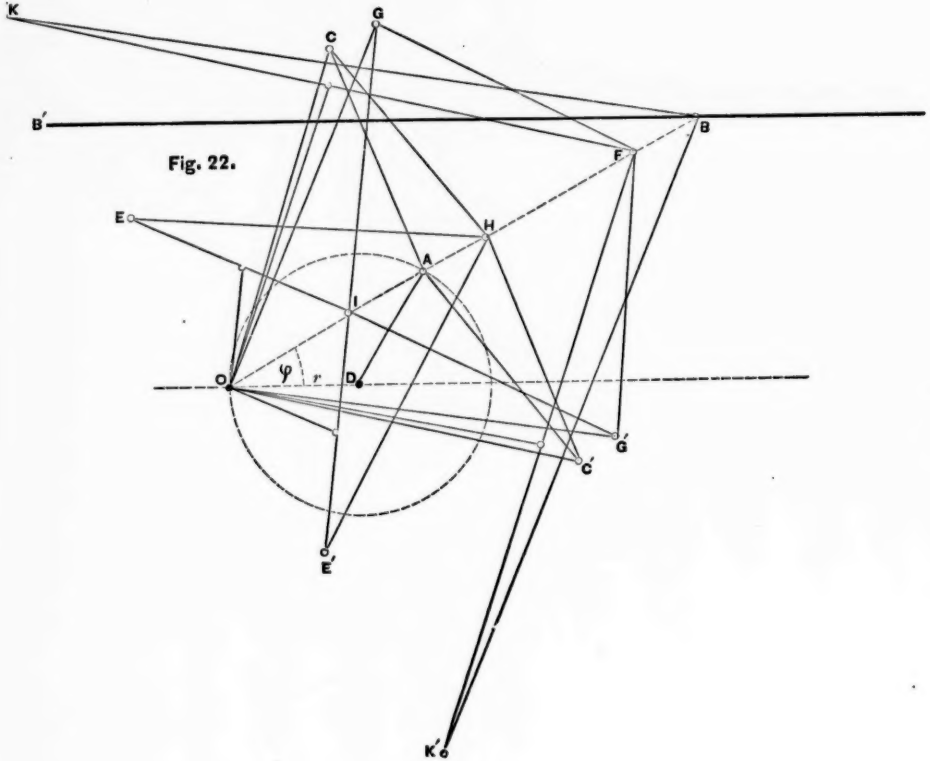


Fig. 22.

Fig. 21.

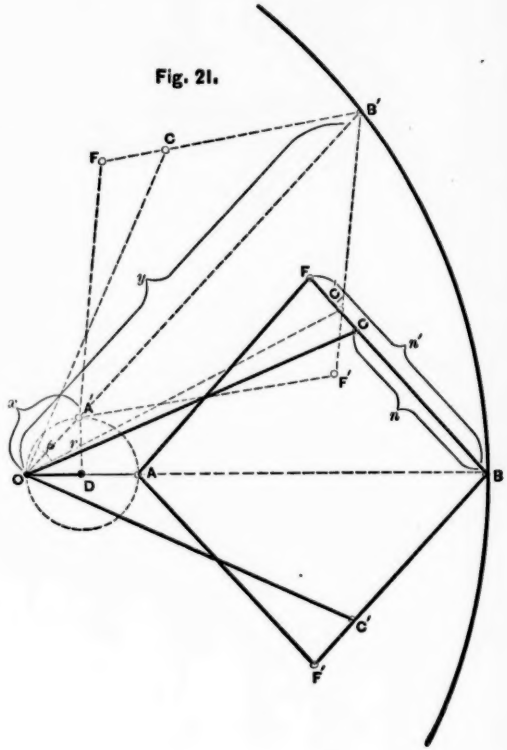
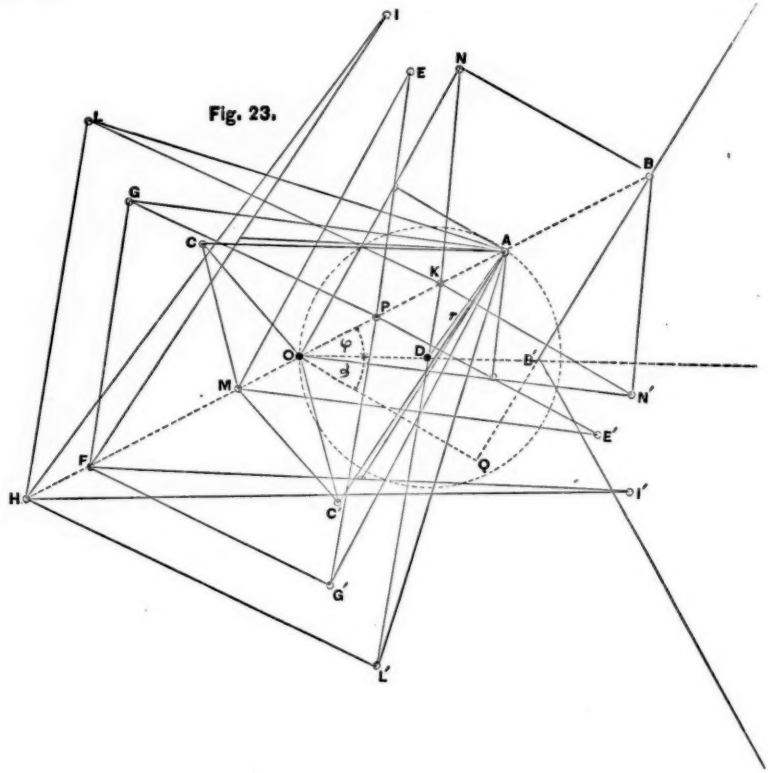


Fig. 23.



$$OB=\frac{C''^2}{2r\cos\varphi-2r\tan a\sin\varphi}$$

$$=\frac{C''^2\cos a}{2r\cos(a+\varphi)}$$

or $$OB\cos(a+\varphi)=\frac{C''^2}{2r}\cos a$$

Now tracing the line OQ making an angle a with OD, and dropping the perpendicular upon OQ from B, then

$$OQ=OB\cos(a+\varphi)$$

and consequently $$OQ=\frac{C''^2}{2r}\cos a$$

Then OQ having a constant value, the line described by the point B is a straight line perpendicular to OQ, and making an angle of $90-a$ with the line through the fixed points.

The distance OB′ is obtained immediately by $\varphi=0$ in the equation which gives OB. This distance then becomes $\frac{C''^2}{2r}$. In the figure C''^2 is taken 1431, $2r=40$ millimeters, and $a=40°$; the apparatus, therefore, describes two sides of a regular hexagon, of which the vertex B is at a distance OB′=36mm from O.

The particular case of a line passing through the fixed points, requires a combination much more complicated, and we believe should be omitted.

If in Figs. 19 and 20 the point O is united to D, we obtain respectively Figs. 24 and 25, of which the bars, eight in number, are movable each in relation to the others, so that any point describes a curve.

Thus the motion of C and B′ is circular relatively to the line which joins O and D; but the motion of any point between C and B′, as E for example' Fig. 26, describes a lemniscate.

The points O, C and C′ describe rela-

tively to A′D arcs of circles, while B gives very remarkable curves which we will study for the particular case of OD=A′D. The point B′, as we have seen above, describes relatively to OD a right line passing always through the point B, Figs. 24 and 25. So that if the rod OD is prolonged and bent to a right angle at the point B, which is the geometrical position of B′ at the moment that A′D coincides with CD, the portion BB′ will pass constantly through B′, whatever the position of the system.

By this means we may lengthen or shorten the consecutive radii vectores of a curve, or in other words, we can derive from a given curve, a trace of the curve lengthened or shortened. This apparatus has therefore been called the *protracteur* or *retracteur*.

Bringing the point H and the two points K and K′ Fig. 27, (situated at a distance *c* from H, in the direction BB′) to the line OD, and describing with H a curve B′A, we shall have the other radii

Fig. 24.

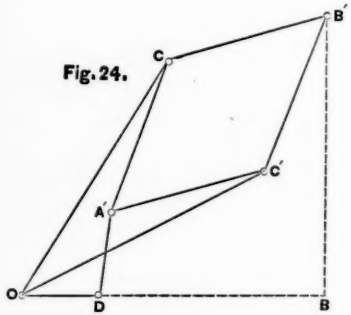


Fig. 25.

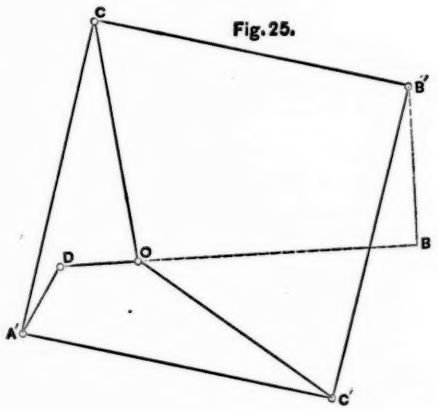


Fig. 26.

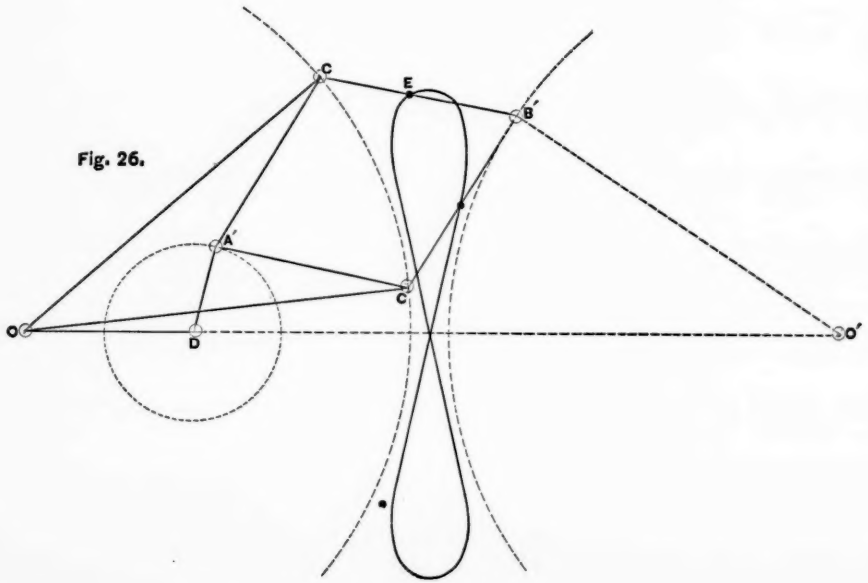
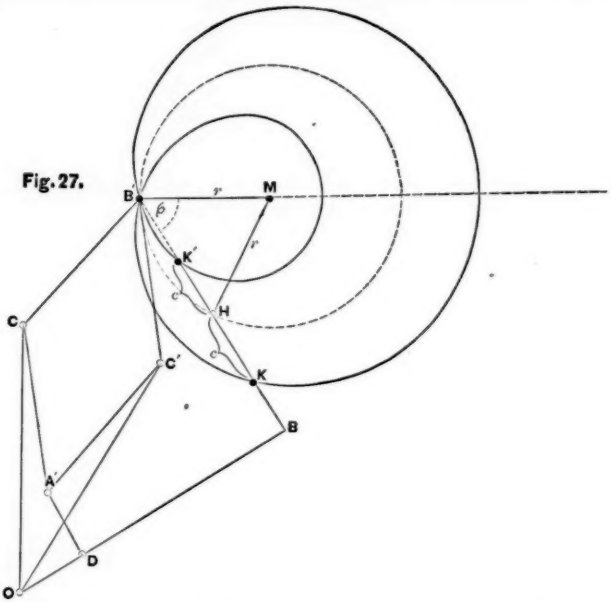


Fig. 27.



vectores, lengthened or shortened by the quantity c.

Thus for the particular case in which the curve is an arc of a circle passing through B′ and of which the center is at M, the curve shortened or lengthened is the *conchoid* with a circular base, known as the limaçon of Pascal.

Its equation is very simple: making the angle $HB'M=\varphi$, and the radius $B'M=HM=r$, we have

$$AK \text{ or } AK'=z=2r\cos\varphi\pm c.$$

For $c=2r$ the conchoid is transformed into the *cardioid*, and for $c=r$ it takes the name of *trimetric curve*. When the point H instead of following an arc of a circle describes a straight line as in Fig. 28, the joining of H to a second system causes the point K to describe the conchoid with a rectilinear base known as the conchoid of Nicomedes, and of which the equation is

$$B'K=z=\frac{B'B''}{\cos\varphi}\pm c.$$

The conchoid with a circular base has been applied by Peaucellier to tracing conic sections. Tracing for this purpose, the radii vectores $BA=2r(\cos\varphi+1)$ of the cardioid or conchoid where $HB=c=2r$ which we have made in this case by means of a negative reciprocator FF′, we have

$$AE=\frac{C^2}{AB}$$

or by substituting for C^2 the two distances EA and AB (for $\varphi=0$); that is to say, substituting the values E′A and AB′ as we have in the following, the equation becomes

$$AE=\frac{AE.'AB'}{AB}=\frac{AE.'4r}{2r(1+\cos\varphi)}=\frac{1}{\frac{1}{2AE'}(1+\cos\varphi)}$$

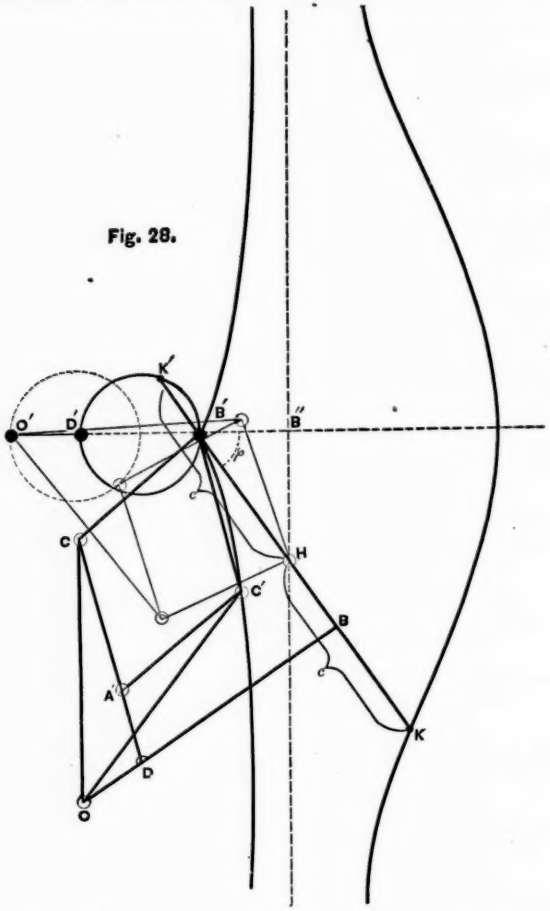
which is the polar equation of a parabola EE′ of which the parameter $p=4AE'$.

For $c=HB$, greater than $AH'=2r$ (Fig. 30) the equation $AE=\frac{AE'AB'}{AB}$ gives

$$AE=\frac{2r+c}{2r.\cos.\varphi+c}.AE'$$

By substituting $AE'=\frac{b^2}{a+e}=a-e$ and $c=\frac{2ra}{e}$ we have

Fig. 28.



$$AE = \frac{b^2}{e.\cos\varphi + a}$$

which is the polar equation of an ellipse whose axes are

$$2a = \frac{2c}{c-2r} AE' = \frac{2AE.H'B'}{H'B'-AH}$$

and

$$2b = 2AE' \sqrt{\frac{c+2r}{c-2r}} = 2AE' \sqrt{\frac{AB'}{H'B'-AH'}}$$

while $e = \frac{2rAE'}{c-2r}$ is the eccentricity.

When on the contrary c=HB is smaller than $2r$=AH' (Fig. 31) we can put $AE' = -(a-e)$ and $c = \frac{2ra}{e}$ and the value of AE is as before $\frac{b^2}{e\cos\varphi + a}$; but this equation is that of an hyperbola of which the axes are

$$2a = \frac{2c.AE'}{2r-c} = \frac{2AE.'H'B'}{AH'-H'B'}$$

and

$$2b - 2AE' \sqrt{\frac{2r+c}{2r-c}} = 2AE' \sqrt{\frac{AB'}{AH'-H'B'}}$$

while $e = \frac{2rAE'}{2r-c'}$

We will now examine the movements of B' in relation to A'D (Figs. 24 and 25).

Conformably with Fig. 32 in which A' and B' have been replaced by A and B,

Fig. 29.

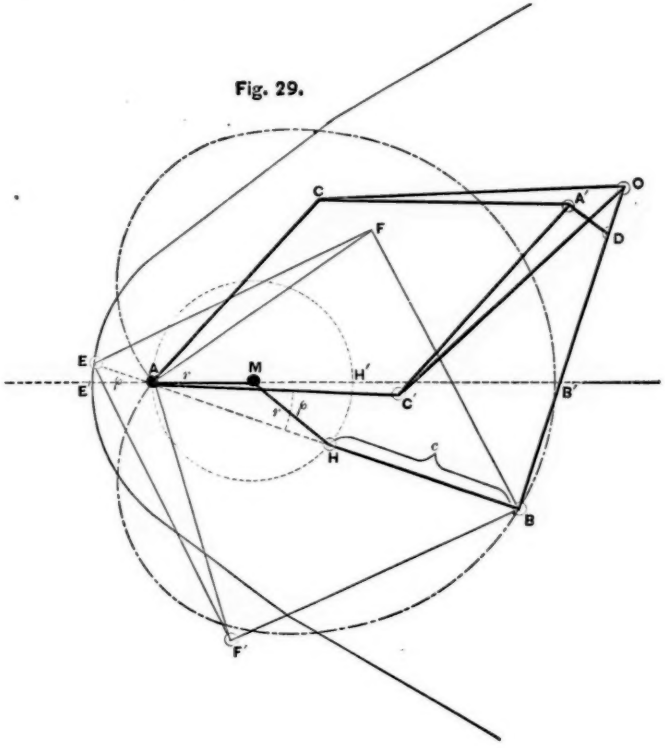
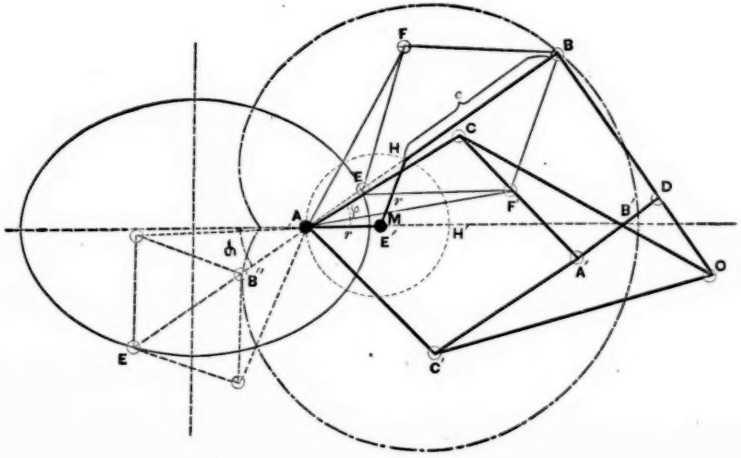


Fig. 30.



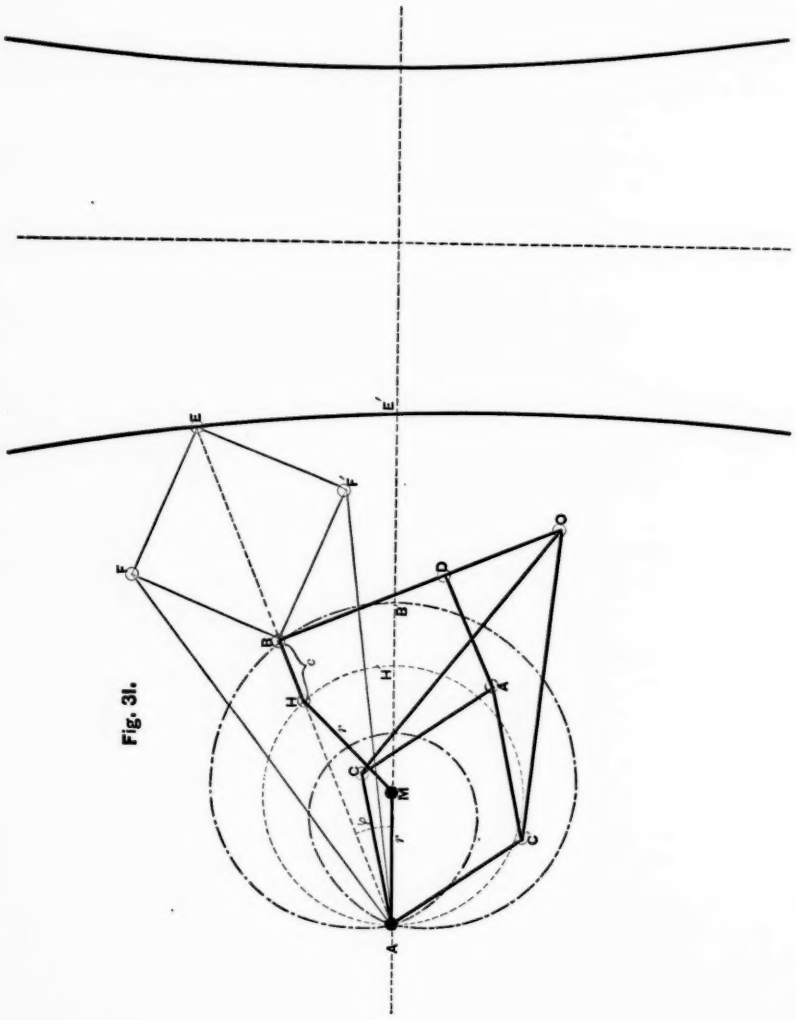


Fig. 31.

the other letters retaining the same signification, we shall have

$$C'=OA(OA+OB)=O'B.'O'A$$

whence (by virtue of $OA=2r\cos\varphi$ and $O'A=2r$),

$$AB=\frac{O'B'}{\cos\varphi}-2r\cos\varphi.$$

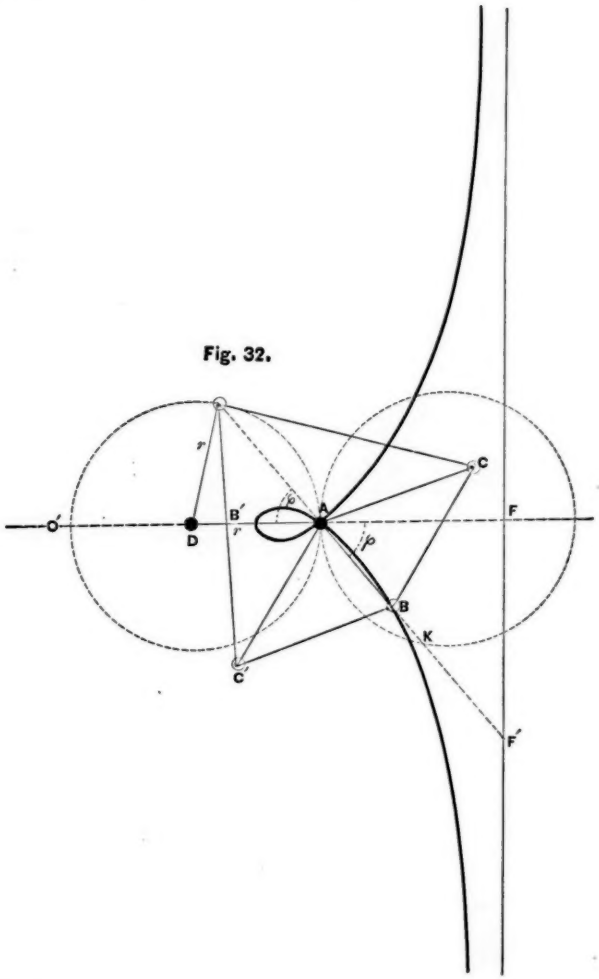
The line FF′ having been drawn perpendicular to AD at the distance AF= O′B′, we have

$$\frac{O'B}{\cos\varphi}=\frac{AF}{\cos\varphi}=AF'.$$

Furthermore $2r\cos\varphi=OA=AK$, and from the supposition that the two circles OA and OK have the same radius, and consequently $AB=AF'-AK=F'K$, from whence it follows that the motion of B in relation to AD is along a *cissoid*.

When $O'B'=2r$, this becomes an ordinary cissoid or cissoid of Diocles, in

Fig. 32.



which the asymptote is at the same time tangent to the circle. And this curve becomes a hypercissoid for $O'B' > 2r$ and a hypocissoid for $O'B' < 2r$.

In seeking for the inverse values of the consecutive radii vectores, we obtain in the first case (Fig. 33)

$$EA = \frac{C^2}{AB} = \frac{\cos\varphi}{\frac{2r\,(1-\cos^2\varphi)}{C^2}}$$

an equation of a parabola of which the parameter

$$p = \frac{C^2}{2r}$$

In the second case we obtain (Fig. 34)

$$AE = \frac{AE'.AB'}{AB} = \frac{AE'(O'B'-2r)\cos\varphi}{O'B'-2r\cos^2\varphi}$$

or putting

$$AE' = 2a \text{ and } O'B' = \frac{a^2}{a^2-b^2}\,2r$$

$$AE = \frac{\cos\varphi}{\frac{a}{2b^2} - \frac{a^2-b^2}{2ab^2}\cos^2\varphi}$$

Then the point E will describe an ellipse whose axes are

$$2a = AE' \text{ and } 2b = 2a\sqrt{\frac{O'B' - 2r}{O'B'}}$$

$$= AE'\sqrt{\frac{AB'}{O'B'}}$$

Finally in the third case where $O'B' < 2r$ we have (Fig. 35)

$$AB = (AK - AF) - OA - \frac{A'F}{\cos\varphi}$$

$$= \frac{2r\cos^2\varphi - O'B'}{\cos\varphi}$$

and consequently the inverse radius vector

$$AE = \frac{AB.'AE'}{AB} = \frac{(AO' - O'B')AE'\cos\varphi}{2r\cos^2\varphi - O'B'}$$

or putting

$$AE' = 2a \text{ and } O'B' = \frac{a^2}{a^2 + b^2}.2r$$

$$\text{then } AE = -\frac{\cos\varphi}{\frac{a}{2b^2} + \frac{a^2 + b^2}{2ab^2}\cos^2\varphi}$$

an equation of an hyperbola having the origin at the vertex, and of which the axes are

$$2a = AE' \text{ and}$$

$$2b = 2a\sqrt{\frac{2r - O'B'}{O'B'}} = AE'\sqrt{\frac{AB}{O'B'}}$$

REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The July issue of "Transactions" is at hand, containing, besides some important discussions of previous papers:

No. 182, On the Flow of Water in Rivers, by De Volson Wood.

THE INSTITUTION OF CIVIL ENGINEERS.—Through kindness of Mr. James Forrest, secretary, we have received the following excerpt minutes of the proceedings:

The Improvement of Dublin Harbor by Artificial Scour, by John Purser Griffith, A.I.C.E.

Strength and Elasticity of Materials, William James Millar.

A Search for the Optimum System of Wheel Teeth, by Edward Sang.

Machinery for the Production and Transmission of Motion in East Lancashire and West Yorkshire, George William Sutcliffe, A.I.C.E.

Experiments on the Filtration of Water, by George Higgin, M.I.C.E.

Dock Gates, by Adam Fettiplace Blandy, M. I.C.E.

We have received also from the same source Part III. of Vol. 57 of "Abstracts."

IRON AND STEEL NOTES.

FOR REMOVING RUST FROM STEEL.—The following mode is recommended:—Place the article in a bowl containing kerosene oil, or wrap the steel up in a soft cloth well saturated with kerosene; let it remain twenty-fours hours or longer; then scour the rusty spots with brick-dust. If badly rusted, use salt wetted with hot vinegar; after scouring, rinse every particle of brickdust or salt off with boiling hot water; dry thoroughly, then polish off with a clean flannel cloth and a little sweet oil.

STEEL RAILS ON THE GREAT EASTERN.—At the recent half-yearly meeting of the Great Eastern Railway Company, it was stated that at the end of the half-year 350 single miles of steel rails would be laid down. The experiments made as to the "life" of steel rails, estimated at thirty years, had been confirmed by the further experience of last year. Circumstances might arise which would render necessary the expenditure on the permanent way up to £150,000. During the next three years the average of 70 miles of steel rails a year would be laid down. A very large amount of steel rails at the present low price was in stock.

THE BRITANNIA TUBULAR BRIDGE.—It was recently stated in several newspapers that the Britannia Tubular Bridge was rapidly rusting away, and that travelers every time they crossed it were placing themselves very near eternity. Mr. Clark, who has examined the bridge, says that the rust of the whole 10,540 tons of which the bridge consists has not in twenty years amounted to a single pound weight; and he adds that if similar care were continued it was impossible to conceive how any practical deterioration could take place in such a structure even in several centuries. Mr. Clark imagines that the tar and sand with which the cells are internally coated, and which is from time to time cleared out, have been mistaken for rust.

UTILIZATION OF BLAST-FURNACE CINDER.—Mr. Franz Büttgenbach, the well-known metallurgist, gives the following method for the utilization of blast-furnace cinder as an insulator for steam pipes, etc.: Mix 150 parts of cinder dust, 35 parts by weight of fine coal dust, 250 parts of fire clay, and 300 parts of flue dust, with 10 parts of cow's hair, add 600 parts of water into which 10 to 15 parts of raw sulphuric acid has been poured, and make a stiff dough of the whole. This is thrown in small amounts upon the warmed pipe, hardening rapidly. Upon this rough coat a second, third, etc., is laid, according to the thickness which is to be used. By the action of sulphuric acid gypsum is formed, and the silica rendered free hardens. The mass becomes as hard as porcelain and is still porous. It adheres firmly and never cracks. Mr. Büttgenbach states that he has tested its merits by ten years' use, and has found it to meet all requirements.

METALLIC ROOFING.—Apropos of new uses for iron, which are now being looked out for with, as it would seem, but insufficient zeal, we may mention that systems of metallic roofing have been for a considerable period in vogue in more than one Continental State. The Montataire Company in France have a patent system of metallic tile and tile fastening, which promises well from the description, and has given, we believe, satisfactory results in practice. A firm in the Harz, at Rübeland, is now making metallic tiles, not like the Montaire tile, of corrugated plate, but of charcoal pig-iron. This firm is said to have been induced to take up the manufacture by noticing that a cast-iron roofing over one of its shops had stood for fifty years without undergoing deterioration. It was also found, at the time when inquiries were made about this, that a neighboring castle, which was the seat of the Dukes of Brunswick, had received a metallic roofing in 1830, and that this roofing was still absolutely sound and weather-tight. The Rübeland Company state that as against common tiles the metallic substitute is lighter in the proportion 35 60. A square meter can be covered for 4*s.* 6*d.*—*Iron.*

IRON AND STEEL OUTPUT.—An elaborate table of the iron and steel output of the chief producing countries in the world has been drawn up by Mr. Commissioner Morrell, which is especially interesting at the present time as indicating the relative position of the producing countries. The percentage of the production may be thus tabulated:

	Iron.	Steel.
Great Britain	46.63	39.70
United States	16.67	26.53
Germ'y (with Luxembourg)	12.16	13.87
France	10.26	10.17
Belgium	4.07	3.61
Austria and Hungary	3.21	4.08
Russia	3 04	.46
Sweden	3.51	.86
Other countries	1.55	.72
	100.00	100.00

The most remarkable feature which is shown by a contrast of these figures for the present time with those of a few years ago is, that the position of the great iron-producing district remains substantially the same, and whilst there has been a very material advance in both that and the production of steel, the United States has increased her output of steel in a much more rapid ratio than have other of the chief producing countries. Whilst Great Britain produces nearly three times as much pig-iron as does America, the output of steel is only 50 per cent. more, and the difference seems likely to be very largely reduced. In the present year, whilst the output of iron is falling in Great Britain, the contrary is the case in the United States; and the production of steel is at its full extent there, whilst here it is below the facilities of production.

RAILWAY NOTES.

EXTRAORDINARY WEAR OF AN IRON RAIL.—At the meeting of the American Institute of Mining Engineers, Mr. W. E. Coxe, of the Philadelphia and Reading Rolling Mills, exhibited an iron rail made at Reading in 1870, which had until 1878 carried 67,000,000 gross tons of freight, cars, and engines, having been worn at the top of the head only 3-16ths of an inch during its nine years of service. The head was made from puddled iron bars, piled, heated, and rolled into bars, 4½ and 3 inches wide by 1 inch thick, these bars breaking joints, heated, and rolled into slabs 9 inches wide and 2 inches thick; the balance of the rail pile, which is in section 9 inches square, was made of 4½ and 3-inch bars, rolled from two-thirds old rails and one-third puddled iron. This was heated and bloomed, and then re-heated before rolling in two high rolls into the finished rail. The test pieces from the head bar of this rail gave a tensile strength of 63,000 pounds. The borings from the head of the worn out rail analyzed as follows:

Phosphorus	.422
Carbon	.027
Silica	.392
Sulphur	.032
Manganese	.164
Iron	98.963
	100.00

SIGNALLING ON RAILWAYS.—Any mechanical invention tending to lessen the risk of mishaps on railways will always be regarded by the traveling public with the hearty sympathy, which a matter of such importance to them can scarcely fail to excite. It is to be feared that notwithstanding the precautions which are taken on most of our great railway systems to protect the public, accidents will often occur from causes over which the officials can have no control. In order to reduce these disasters to the smallest possible compass, some experiments with what was termed a "chemical signal fire" were undertaken the other evening at the Elmers End station of the South-Eastern Railway, by Mr. Varley, of Norwood. The new signal is produced by a torch, which, when ignited at the top, throws such a light across the railway as would from its brilliancy immediately attract the attention of an engine-driver who would have ample time to stop his train before reaching the scene of the accident. From an elevated post the signal could be seen about a mile-and-a-half off. The torch burns for about five minutes, and by its aid a guard, in case of a break-down or collision, could warn the drivers of approaching trains of their danger. Mr. Varley stated that the Abbots Ripton and Hatfield accidents might have been prevented had this signal been available, as in each case the guards were unable to give notice of the disaster because of their lamps becoming useless. One of the torches was ignited at the Woodside Station, at a curve of the line about a mile off, and though the light itself could not be seen, the reflection from it would be sufficient to attract the attention of the most careless observer.

A NEW ALPINE RAILWAY.—On July 25th the southern portion of a new Alpine railway, to connect Italy and Austria, was opened at Ponteba, in the Venetian district of Udine. The ceremony had, for reasons to be referred to presently, no official character beyond the presence of some of the authorities of the "Alta Italia" line. But there were many visitors from Udine and even from distant Venice. An improvised railway terminus was gaily decorated. A military band played all day, and even all night; for there was a grand ball, although somewhat shorn of its splendor owing to the absence of the Austrian engineers, who had constructed all the northern and by far the largest portion of the line. Yielding to exaggerated national feelings, the Italian managers had removed the Austrian flags, which at first formed part of the decorations; and the Austrians felt constrained to absent themselves, lest they should seem to acquiesce in what may have been intended by some as an affront. The new line, which runs from Travis by Pontafel and Ponteba to Udine, will, when it is altogether opened for traffic, be a most valuable addition to the facilities of communication between Austria and Germany and Italy. It is enough to say that it will shorten the journey by seven or eight hours. It will give Italy direct access to the forests and iron mines of Carinthia, for the mutual advantage of both regions. It also opens up an easy road for the transport of corn from Hungary to the northern Italian seaports. The journey from Villach to Udine can be performed in four hours; five hours more will take the train to Venice. In the other direction, the journey from Villach to Vienna will be performed in twelve hours. Thus Vienna will be brought within twenty-one hours' distance of Venice, instead of twenty-eight as at present. The portion of the line which lies within Austrian territory has been for over six months completed. But owing to differences, partly political, partly administrative and finanical it cannot yet be opened for traffic. In the original arrangements between the Italian and Austrian Governments certain rates, both for passengers and goods, were agreed upon; but the Italian authorities now insist on changes which the Austrians consider would be very prejudicial to their own interests. Again, it was agreed that there was to be a common Custom house at Pontafel, and the station there was built of dimensions capable of giving the necessary accommodation. Now the Italians demand that the Custom house shall be on Italian territory at Ponteba, where there is not as yet even a regular railway station. The Austrian companies, also, over whose lines the through traffic should pass, have not been able to come to an agreement as to the proportion each should have out of the traffic charges. Finally, that part of the Italian position which joins on to the Austrian piece near Pontafel, has not yet been quite finished; although the original contract fixed October last year for the date of completion. The Italian portion, which was opened on July 25th is admirably constructed. There are very many tunnels and bridges. Some of the works can hardly be paralleled on any other European line. The general scenery resembles that of the Gothard valley, from Fluelen by the Devil's Bridge to Hospenadi.

CONVEYANCE OF TROOPS BY RAIL.—A novel experiment was recently tried at the Aldershot Railway Station, with a view to testing the time necessary for a half battery of Royal Horse Artillery to load and unload on and from a train in the presence of an enemy. It was assumed that a force advancing on Aldershot had part possession of the South-western line, and that it was necessary to hurry up artillery to support infantry in checking its further advance. For this purpose a half battery, full war strength, composed of three guns, three officers, eighty-eight men, and eighty-eight horses, under the command of Captain England, were ordered to report themselves at Aldershot Town Station at nine o'clock. On arrival they found in the goods yard a train awaiting them, consisting of eight trucks for guns and carriages, eleven cattle trucks for horses, one horse box for officers' chargers, and two carriages for officers and men. There was no difficulty whatever about the loading operation, which took place from a platform almost level with the embarking troops, and was accomplished in the short time of forty-five minutes. The train was then shunted on to a siding, it being supposed that having traveled a certain distance it had found the track torn up, and that the exigencies of the occasion required an immediate landing at a point where there were no immediate appliances at hand for the disembarkation of troops. The "down" as well as "up" having been destroyed, it was suggested by Mr. Adkin, the Aldershot station master, that rails should be used for the purpose of unloading the guns. A portion of the line was therefore supposed to be torn up and laid side on to the end of the gun trucks, the guns and carriages being lowered by means of ropes on the permanent way. Then arose a point as to how the horses were to be disembarked from the side of the cattle trucks. There was no hesitation about the matter, spare sleepers having been brought into requisition they were piled on top of one another, so as to form a stage, and on them were placed a couple of rails, over which other sleepers were thrown, so as to form a good solid way over which horses could travel. As each truck was discharged the train moved on, so that truck by truck the horses were disembarked. The whole operation was conducted without a hitch of any kind in the short space of one hour and twenty-five minutes. This was rather a severe trial to the men, as in actual warfare they would have been assisted by infantry, and doubtless have gone through their work in much less time. The result of the trial was considered to be highly satisfactory.

THE PAST, PRESENT, AND FUTURE OF RAILWAYS.—In a recent publication, Sir Edward Watkin reviews the past, present, and future of English railways. The contrast between the railway system of the 14th of September, 1830, when the Liverpool and Manchester line was opened, and that of to-day, is great. In 1830 a total of 55 miles of railway was tra-

versed by locomotives; now there are 17,500. Then the speed of trains might be taken at a maximum of 20, now of 60 miles an hour. Then the weight of the engine was under 6 tons, now under 40; while per ton of weight the modern engine is much more powerful. The capital of the Liverpool and Manchester Railway, of 31 miles, was, according to prospectus, £400,000, and that of the other railway —the Stockton and Darlington—probably £200,000 more. Now there is an expended capital of about £700,000,000, and an annual gross revenue of £65,000,000—in each case approximating in amount to the debt and annual expenditure of Great Britain. Six hundred millions of passenger tickets are issued each year, or nearly twenty times the number of our population; 400,000 persons are, in addition, regular travelers as holders of periodical tickets. 70,000,000 tons of merchandise and 150,000 tons of minerals are also annually conveyed. As regards the future, Sir Edward Watkin believes the desideratum to be greater speed of traveling and transit; not greater speed probably in every train, but a greater average speed. At present the goods and coal and stopping passenger train is in the way of the fast through express, and some of the greater companies have battled with the difficulty by doubling their lines of rail, thereby expending a large new capital, totally out of proportion to the new money-earning means obtained. It has been found that so much space represents so much traffic-carrying power; not an indefinite quantity. Unless it could be assumed that augmenting trade would not in a while be an aid, these duplications, in the absence of largely increasing receipts, must rapidly tend to reduction of dividend. In fact, it is possible to realize a state of circumstances under which the proportions of capital and net earnings might become so much disturbed that the "ordinary stock" of railway capital would be largely depreciated. Capital is an increasing quantity; working expenses are an increasing quantity also, and taxation and compensation show the same tendencies. These are conditions that stand in the way of great improvements of service; for a poor railway can do nothing but hold on. Too many railway managers, for want of a thorough experience "out-of-doors," do not comprehend that the secret of dividend is minimum of capital outlay and the maximum of traffic at paying rates. The chief object is the maximum of utilization of railways. If a passenger can travel at sixty miles an hour, the reason why mineral trains cannot be equally expeditious is because proper vehicles are not constructed for the purpose, and the result is only a partial user of lines of road. Sir Edward Watkin would have all plant alike made fit for running at a higher speed. Much as England owes in the past to railways, in the future a great deal more will be expected from them. The railway is the only shop which cannot shut its doors. It must, almost without notice, take up and set down as many passengers as choose to present themselves at thousands of booking offices, or as many tons of goods and minerals as the owners see fit to transmit. In the economy or "the exhaustive use" of time lies the secret of success or failure individually and nationally.

ENGINEERING STRUCTURES.

IT has been announced in the *Temps* that the boring of the shaft for the purpose of investigating the nature of the chalk strata through which the submarine tunnel between England and France is to pass, was resumed on the French coast at Sangatte on the 1st of March last. The depth of the shaft, which at the end of last year was at 34.35 meters, was extended to 38.50 meters—that is, to a depth corresponding to 8.67 meters below the low water level. At this depth the flow of water into the shaft amounted to 1300 liters per minute, and the pumps became insufficient. They are to be replaced by others of a more powerful character.

NEW STEEL RAILWAY BRIDGE.—A new railway bridge over the Missouri river, built wholly of steel, has lately been completed and opened for traffic by the Chicago and Alton Railway Co. The bridge is located at Glasgow, Mo. The constructing engineer was Gen. Wm. Sooy Smith. The material was furnished by the Hay Steel Co., of Chicago, and while the structure is stronger than an iron bridge its weight is thirty-three per cent. less. The time of construction was only one year. The cost, $450,000. The following are the principal dimensions:

Five spans, 314⅔ feet each, from center to center of piers, three above and two below grade; all steel; depth of truss, 36 feet center to center of pins. Height of through spans above high water, 50 feet. East approach, iron trestle, 210 feet; two deck spans of iron, 140 feet each, 280 feet; west approach, iron deck span, 140 feet; west approach, iron trestle, 510 feet; west approach, wooden trestle, 864 feet; total length of the bridge proper (steel), 1,573⅓ feet; total length of bridge and approaches, 3,577⅓ feet.

THE Solihull sewerage works, which consist of a sewage farm, situated near Catherine-de-Barnes, Solihull, and between three and four miles of sewers, were officially handed over on the 29th ult. to the Solihull rural sanitary authority by the contractors, Messrs. W. Green and Son, of Warwick. The works, which have been designed by Mr. E. Pritchard, C. E., Warwick and Westminster, and carried out under the supervision of Mr. C. Lewall, C. E., the resident engineer, were commenced in July, 1878 and have cost between £6,000 and £7,000, provision being made for a population of 7,000. The sewers are upon the straight line system, with man-holes and ventilators at each change of inclination, both vertical and lateral. The length of sewers is three miles 410 yards; viz., 2,590 lineal yards of outfall 15in. diameter, and 3,100 lineal yards of branch sewers 9in diameter. The depth of sewers, in consequence of the superficial configuration of the ground, is great, being nearly 20ft. in places. At each end of the sewers is a flushing-chamber, containing between 300 and 400 gallons of water. The sewage farm is 15 acres in extent, and has been obtained on lease for twenty years at a rent of £4 per acre. The mode of disposing of the sewage is filtration and irrigation combined. The sew-

age is first received into tanks, which are in duplicate, where deposition takes place, and subsequently it is clarified by upward straining through a bed composed of 9in. of thick gravel. The clarified sewage then passes through underground carriers, and is distributed over the ground by means of brick chambers of a novel character, supplied with disc valves. The land is divided into four plots, and is thoroughly drained to a depth of about 6ft. For times of considerable rainfall provision is made by beds three-quarters of an acre in extent, where, by intermittent filtration, the sewage is disposed of.

RAILWAY ACROSS THE SAHARA.—Mons. de Freycinet, Minister of Public Works, lately presented to the President of the French Republic the report of the committee appointed to consider Mons. Dapouchel's project for uniting Algeria with the Niger by an unbroken line of railroad 1,250 miles in length. The general statement of the committee was to the effect: 1st. That there exists in Soudan a large population, a fertile soil, and natural riches which are uncultivated. It is very important to open outlets for commerce through the French possessions, which are the most favorably situated for this purpose. France ought to follow the example of England, and do her best to induce the caravans to cross French territory instead of only coming to its borders. 2d. The opening of a railway joining our possessions in Algeria with Soudan is necessary in order to obtain this double result. 3d. It is also necessary to join Senegal with the Niger. 4th. The explorations or surveys of the undertaking ought to be directed simultaneously from Senegal and from Algeria, and the plans ought to include both directions. 5th. South of Algeria, the uncertainty which exists regarding the topography, the climate, the resources, and the inhabitants of certain parts of the Sahara, make it necessary to proceed with care in order to avoid mistakes and military complications. 6th. It will be advantageous to start immediately with a preliminary line between Biskra and Ouargla of about 200 miles. This line can be joined by the Hodua to the line from Algiers to Constantine. As far as Ouargla ordinary escorts appear sufficient to protect all operations. 7th. A credit of £800 will be demanded to meet the expenses of the surveys, and the needs of the explorers.

While the committee have been considering the matter referred to them, committees of the Chambers of Deputies and the Senate have expressed decided opinions on the need of immediate action in regard to the opening of railways in Northern Africa. The Commission of the Senate on the Railroads of Algeria have reported as follows:—"The committee unanimously agree to the surveys which are necessary for the execution of the railroad across the Sahara; they are certainly convinced that it is greatly to the interests and to the advantage of our country for us to solve this problem of the basin of the Niger, and to be in advance regarding the civilization of those countries which our colony of Algeria opens to us."

At the suggestion of M. de Freycinet, the President of the Republic has appointed a Commission, with power to cause surveys to be made, and to institute such exploring expeditions as are found to be necessary to decide as to the practicability of constructing the proposed railroad. Those interested in the scheme are of opinion that the Sahara itself will not form so serious an obstacle as has been supposed, and that it will be possible to obtain water by sinking wells in many parts of the desert district.—*Eng. Mech.*

ORDNANCE AND NAVAL.

UTILIZATION OF OLD GUNS.—A great number of breech-loading guns, Armstrong pattern, 25-pounders and 40-pounders, which have lain unused, in the Woolwich Arsenal, for a good many years, are now being brought forward for employment, and, after examination and any necessary repairs, they will be mounted in Cork Harbor, and such-like places, for coast defense. Many of these guns have never left the arsenal since they were made, and the remainder, though classed as repairable, have been scarcely thought of for ten years past, until the reaction in favor of breech-loaders has once more brought them into notice.

THE CHINESE GUNBOATS.—The trial of four gunboats, supplied by Sir William Armstrong & Co. to the Chinese Government, took place yesterday. These vessels are called the Epsilon, Zeta, Eta, Theta, in succession to the four previously sent out. The trial was witnessed by the Admiralty and War Office authorities, as well as by the Chinese ambassador and suite. The gunboats steamed down Spithead and fired their heavy guns both with blank cartridge and Palliser shot. Each boat carries one 35-ton gun, with an enlarged chamber, which gives it much greater power than the 38-ton gun at present in our service. At the conclusion of the trial, which was very successful, the boats started together for Plymouth, *en route* for China.—*Builder.*

ARMOR-PLATE EXPERIMENTS AT PORTSMOUTH.—Another highly satisfactory trial of a compound steel and iron armor-plate, intended for the protection of the turrets of the double turret ship Inflexible, and manufactured by Messrs. Cammell & Co., of Sheffield, was made on board the Nettle, target ship, at Portsmouth. The test was of the same crucial character as on former occasions. Three rounds were fired from the 12-ton 9-inch gun at the plate at spots a foot apart, the range being 30 feet, the charge, 50 lbs., and the weight of the Palliser projectile, 250 lbs. The manufacture of this novel description of composite armor has been brought to so high a state of perfection, and the ordeal through which each plate is, for prudential reasons, required to pass previous to its acceptance by the Admiralty, is now attended with such uniform results, that the effect of each impact might be well-nigh foretold. The plate which was, on this occasion, subjected to trial was the fourth of the seven which had been manufactured by the Sheffield firm for the fore turret, and the extraordinary resistance offered to the penetration

of shot exhibited by the armor was so like that of previous plates that, with the alteration of a few details of no importance, the description of one series of tests would apply to all. In every instance, the projectile broke up and fell into a *mitraille* of splinters upon the gun deck, the splash of the projectile upon the steel face of the shot opening a crack round the lower curvature of the impact and filling up the indent with its own semi-molten metal. The first round struck 2 feet 6 inches from the right edge of the target and 4 feet 7¾ inches from the bottom edge, producing three radial cracks of very regular form, one of which extended to the right edge. This was substantially the same as happened with No. 3 plate, and the diagram or picture taken after the first round, on that occasion, was almost a *fac-simile* of the appearance presented before. The second shot struck 5½ inches to the right of the former point of impact and 23 inches below it. It opened some very fine hair cracks, which could just be observed underneath the film of paint with which the surface of the plate had been covered for photographic purposes. It also produced two cracks of no apparent depth at spots remote from the part hit, and which were probably caused by vibration. One of these extended to the right edge. The third round—always the most destructive because of its cumulative effects, and also in consequence, no doubt, of the movement set up in the molecules of the plate itself—hit 15½ inches below and 16 inches to the left of No. 1. This blow opened four more cracks, one extending from the point of impact to the left edge of the plate. The punishment, in itself, was slight, but the force of the blow developed some of the radial cracks produced by the previous rounds until they seemed to have been produced by the last shot. Many of the cracks extended only slightly below the surface, and, in no instance, did any of them, when fully developed by subsequent impacts, sink further than the weld; and so far was the plate from being penetrated by the projectiles that there is reason to believe the indents did not proceed further than the iron backing which holds the steel together. These consecutive trials on board the Nettle are gradually proving that the use of steel, in combination with iron, for defensive armor will be the best means of keeping out shot and shell by breaking them up on impact.

A Great Gun Trial at Woolwich.—The first shot was fired from the first of the 100-ton guns at Woolwich recently, and was the occasion of much interest. It may be necessary to explain that, at the time when the special vote of credit was passed, in the spring of 1878, it was discovered that Sir William Armstrong had on hand, at his Elswick factory, four guns of 100 tons apiece, and that their ultimate destination was too doubtful to permit of their being disregarded, one such gun being calculated to destroy any armored ship at present afloat. It was determined, therefore, to purchase them, and they became the property of the British nation at the cost of £16,000 per gun, when completed. The four guns have now been delivered at the Royal Arsenal, and it was one of them which was put to the proof. Its conveyance to the proof butts below the Arsenal was a source of some anxiety, for it is questionable whether so heavy a deadweight, standing upon so limited a base, has ever before been taken along a railway, and neither the rails to the butts nor the bridge over the canal were designed for such a task. The Royal Engineers had, however, fortified the bridge with two additional girders and a packing of wood and iron, and the rails, being mostly steel, were trusted to do the work. Two locomotive engines, the "Gunner" and the "Driver," belonging to the Royal Gun Factories, sufficed for the duty, a third engine, borrowed from the Southeastern Railway Company, being not required. Instead of being on a low sleigh, as most guns proved at Woolwich are, this gun stood elevated upon a superstructure consisting of its own service carriage, and the slide upon which it recoils, the whole mass resting upon two little six-wheeled bogie trucks, making the total weight 170 tons. In the eyes of the skilled engineers of the Royal Arsenal, the passage of the gun was a grand procession, and the gun on its car, representing an outlay of £25,000, seemed to embody the triumph of mechanism. The canal was safely crossed, the bridge giving barely three-quarters of an inch under the enormous load, and recovering its shape immediately afterward. Preparations were immediately made for the trial, but this was delayed until five o'clock for the expected arrival of a member of the Armstrong firm, who, however, did not appear. The gun stood for the ordeal upon the inclined rails from which the 81-ton guns were fired, and about 80 yards in front was one compartment of the butts, a bay filled with sand, which was to receive the shot. The shot was a flat-headed projectile, weighing 2,010 lbs., or just about 18 cwt. Like the gun and carriage, it was made at Elswick, and was fitted with a gas check, which is now adopted, generally, instead of studs, to follow the grooves of the rifling. It was made very little less than the bore, which has a caliber of 17¾ inches, increasing to 19¾ inches in the powder chamber. The thickness of metal at the muzzle is about 5 inches only, but at the breech end the chamber is surrounded with a wall of iron 2 feet 5 inches through, making the maximum diameter 6 feet 6 inches. In length the gun is 36 feet, of which the bore occupies 33 feet, and the total length of gun and carriage, when run out for firing, is 44 feet. The experiment was conducted under the supervision of General Younghusband, R. A., the head of the Gun Factories, and president of several scientific committees. Captain Morley, R. A., proof master, directed the loading; and among the other officials engaged were Colonel Noble, R. E., Inspector of Works; Major F. Ellis, Secretary of the Ordnance Committee; Captain O'Brien, R. E., and Mr. R. S. Frazer, Deputy Superintendent of the Royal Gun Factories. The cartridge, consisting of 440 lbs. of cube powder, strongly bound in canvas, and stiffened by wooden bands, was rammed home, occupying 5 feet of the bore, and then followed the projectile, the length of which was 2 feet 8 inches. Electric screens were placed at intervals, to measure the veloci-

ty as the shot passed through them, and crusher gauges were inserted both in the base of the shot and in the base of the chamber, to record the pressure of the explosion in those situations. The spectators having retired to a prudent distance, the gun was fired, by electricity, from the instrument room, and recoiled a good way up the platform, but suffered no damage either to itself or carriage. The screens registered a velocity of 1,590 feet per second, which is a good figure, but the projectile was found to have broken up, which may have affected the result. The fragments of the shot and the expansion of the sand did considerable damage to the wooden enclosure of the butts, which are, however, in a dilapidated condition.

The experiments were to have been concluded on Monday, as the drawbridge is immovable until the return journey is accomplished, but the unexpected results of the one shot fired on Friday have decided the authorities to take time for consideration. It is understood that an examination of the crusher-gauges placed in the gun has revealed a high degree of pressure, which is scarcely consistent with the conditions, and hence there is a pause before proceeding to higher charges. When it is remembered that 25 tons to the square inch was the safety limit in the case of the 81-ton gun, it may appear that there is nothing remarkable in a mean pressure of about 20 tons to the inch in the case of the 100-ton gun; but there are other facts to be included in the comparison, one of which is, that the powder charge, although weighing about 4 cwt., was, by no means, large for such a gun—less, in reality, by 10 lbs., than the maximum charge fired by the Woolwich gun last year. Again, the altered circumstances arising from the unusual shape of the projectile, which was short for the gun's length, and presented considerable area of resistance, are quite at variance with previous experiences in British gunnery, and preclude the possibility of fair comparison. Another reason for the postponement is believed to be the breaking up of the Armstrong shot when fired into the butts. It is not taken for granted that this failure was due to defective manufacture, for the service projectiles, however carefully made, have sometimes split into fragments before or after leaving the gun, and, formerly, proof shot was especially liable to this weakness, being cast with less caution than at present. It is conjectured that the breaking up was caused by the projectile striking against another lying buried in the sand of the butts, and it is asserted that, even if it parted immediately upon leaving the gun, there would be little possibility, at so short a range, of any of the fragments going over the butts. There are, however, two more projectiles of the same workmanship to be fired before the proof is completed, and, as it is arranged that the three other 100-ton guns shall be subjected to the same tests, the bad luck of the first shot has occasioned some misgivings. The gun has been examined by taking impressions of the bore, and, as far as can be ascertained by this test, it was found to be no worse for the preliminary ordeal through which it has passed.

—*Iron.*

BOOK NOTICES.

AN INTRODUCTION TO THE PRACTICE OF COMMERCIAL ORGANIC ANALYSIS. By Alfred H. Allen, F. C. S. Vol. I. Philadelphia: Lindsay & Blakiston. For sale by D. Van Nostrand. Price $3.59.

The extent of the ground covered in this first volume is briefly expressed by the mention of the groups: The Cyanogen Compounds, Alcohols, Neutral Derivatives of Alcohol, Acid Derivatives of Alcohol and Vegetable Acids, Phenols and Acid Derivatives of Phenols.

One characteristic of the work is an absence of symbolic abbreviations, and which will be regarded as a merit by many. The typography is good.

MODERN METEOROLOGY: A Series of Six Lectures before the British Meteorological Society. London: Edward Stanford. For sale by D. Van Nostrand. Price $1.50.

These lectures are by eminent men, but are prepared for audiences familiar only with the common and obvious properties of the atmosphere, and with only the leading laws of physics. The book may be profitably read by the tyro in science with pleasure and much profit.

The lectures, in order, are:

The Physical Properties of the Atmosphere. By Robt. James Mann, F. R. A. S., F. R. G. S.

Air Temperature; Its Distribution and Range. By John Knox Laughton, F. R. A. S.

The Barometer and its Uses, Winds and Storms. By Richard Strachan, F. M. S.

Clouds and Weather Signs. By Rev. W. Clement Ley, F. M. S.

Rain, Snow and Hail. By George James Symons, F. R. S.

Methods and General Objects of Meteorology. By Robt. H. Scott, F. R. S.

FIRST BOOK IN QUALITATIVE CHEMISTRY. By ALBERT B. PRESCOTT, Professor of Applied Chemistry in the University of Michigan, Author of "Outlines of Proximate Organic Analysis," "Chemical Examination of Alcoholic Liquors," and joint-author of "Qualitative Chemical Analysis." New York: D. Van Nostrand. Price $1.50.

The following is from the author's preface:

"This little manual has been prepared for certain classes who take a short course in qualitative chemical work, to attain some practical acquaintance with the materials of every-day life, rather than to qualify as analysts. The scope of the work includes a more definite study of bases and acids than is taught in ordinary courses of experimental chemistry, and a broader study of chemical characteristics than is provided in the common rudimentary qualitative analysis. * * Its limits as a first book are further explained by the circumstance that it is prepared, primarily, for use under teachers who also employ the author's larger work upon Qualitative Analysis."

The success of Prof. Prescott as an instructor and writer will insure a good reception for this elementary work.

REPORT ON A PRELIMINARY INVESTIGATION OF THE PROPERTIES OF THE COPPER-TIN ALLOYS. By ROBERT H. THURSTON.

Washington: Government Printing Office. For sale by D. Van Nostrand. Price $2.50.

This is a report upon one branch of the labor assigned to the United States Board, which was established to direct a series of tests upon iron, steel and other metals. A committee, formed by the Board, were directed to determine the mechanical properties, the physical and chemical relations of alloys of copper, tin and zinc. The researches were conducted, under direction of the chairman of the committee, in the Mechanical Laboratory of the Stevens Institute.

The experiments upon alloys of copper and tin are alone reported in this work. A similar report will include the investigation upon the copper-zinc alloys, and still another will be devoted to the triple combination.

Prof. Thurston, the chairman, expresses the hope that this preliminary study will prove to have been so satisfactorily done that a repetition of the work may never be required, and that, in the future, research may be confined to matters of detail in those parts of this field which are here shown to be of most promise.

As this work shows the results of the only systematic and thorough researches upon these alloys, it will everywhere be regarded as of great value.

THE SILK GOODS OF AMERICA. By WM. C. WYCKOFF. New York: D. Van Nostrand. Price $1.50.

The growth of the silk industry in this country, during the past few years, has developed to proportions of which the general public know comparatively nothing. Every one knows that both foreign and domestic silks are cheaper now than formerly, but few people know that American goods are better as well as cheaper. Correct information is withheld, in many cases, by the practice, steadily pursued in the retail trade, of representing the entire product of some of our silk mills as of European make.

The author of this work thinks it is time that the merits of American silks should be made fully known to the American public.

The first portion of the book is devoted to an account of the processes of manufacture, divided as follows:

Chap. I.—Introductory; II.—Raw Silk; III.—Sewings and Twist; IV.—Weaving: Preparatory Processes; V.—Black Dress Goods; VI.—Piece Goods; VII.—Spun Silk; VIII.—Handkerchiefs, &c; IX.—Ribbons; X.—Trimmings, &c.; XI.—Silk Laces; XII.—Dyeing.

The remainder of the work is statistical, but the whole is of interest to all who take pride in the progress of American manufactures.

CABLE-MAKING FOR SUSPENSION BRIDGES, WITH SPECIAL REFERENCE TO THE CABLES OF THE EAST RIVER BRIDGE. By W. HILDENBRAND, C. E. New York: D. Van Nostrand. Science Series. Price 50 cts.

Some of the most useful books for young engineers are those describing in detail the construction of special works, giving the reasons for the selection of the various parts, and the calculations determining the section, strength, material and position of those parts. Such a work is the one before us. The author describes the work as on the manufacture of wire cables after the method invented by John A. Roebling, and followed by him and his sons in making the largest suspension bridges in America, and, particularly, he describes all the points, including the theoretical investigations for determining the sizes of the cables for the suspension bridge connecting Brooklyn with New York. This bridge has a floor 85ft. wide, is supported by four cables suspended in three spans, the middle span being 1595.5ft. between the centers of the towers, the side spans 954.5ft. from towers to points of cable attachment. Each cable consists of nineteen strands of 332 parallel steel wire, thus containing a total of 6,308 wires, with a total ultimate strength of 10,730 tons. All kinds of traffic, including railway cars drawn by endless ropes, pass over the bridge. The cables rest upon the towers at a point 267ft. 6.5in. above high-water mark. The lowest point of the bridge in the center of the river "shall, in no case," says the law, "be less than 135ft. above mean high water." All the auxiliary apparatus for getting the cables into place; the traveling ropes and foot-bridge stays, saddles, anchorages, effects and methods of meeting changes of temperature, are carefully, though briefly, described in this little book. It will, at once, be seen that a description of this work as we have above sketched out is valuable, and readers will find that Mr. Hildenbrand has dealt with the subject in a clear and simple manner, never forgetting to state, when possible, the why and the wherefore of this detail, or that alteration of plan, keeping his calculations well within the grasp of those who have a fair knowledge of algebra and analytical conics. The little book is one of a series which includes several of a useful elementary character.—*Engineer.*

MISCELLANEOUS.

THAT steamships are superseding sailing ships very rapidly, both for home and foreign trade, becomes more apparent with every fresh publication of Board of Trade statistics. In the foreign trade alone steam tonnage shows an increase of 10 per cent. for 1879 as compared with last year, whilst a corresponding decrease is noticeable in sailing ships employed in the same trade.

FOR the New Jersey ship canal to unite the Passaic and Hackensack rivers with the Hudson, and effect a saving of about twelve miles by water between Newark and New York, three routes are reported to be under the consideration of General Newton, to whom the Government has entrusted the duty of making the surveys. One would pass directly from Newark Bay through Bergen Hill by a cutting of 4,732 ft.; a second would take a diagonal course through the hill, and save about 250 ft. of cutting over the first; and the third would follow a straight line from Newark Bay, near the beacon light, to a point midway between the two bays, where they intersect the Morris Canal, and thence follow the line of the canal to New York Bay.

THE digging of a canal from Cronstadt to St. Petersburg is making such progress that Admiral Possiett, who directs the work, assures the Government that in a year's time vessels of small size will be able to pass from the sea to the Neva, and that, in the summer of 1881, the canal, the depth of which is fixed at 20 feet, will have been excavated to the extent of 16 ft., enabling a goodly-sized craft to reach the capital.

THE Government of Victoria have obtained from the Local Legislature an Act authorizing them to hold an International Exhibition, upon a somewhat extensive scale, at Melbourne. The building, which is to cost about £95,000, is in course of erection, and the exhibition will be opened on October 1st, 1880, and closed on March 31, 1881. Information on the subject may be obtained from Mr. George Collins Levey, C. M. G., Secretary to the Commissioners, 8 Victoria Chambers, Westminster.

HERR TH. FLEITMANN has published a paper upon the malleability of nickel and cobalt. He proceeded upon the hypothesis that this want of malleability might be ascribed to their absorption of carbonic oxide during the process of their preparation; and, knowing the readiness with which metallic magnesium, under certain conditions, will destroy the oxides of carbon, he fused the metals nickel and cobalt with a small quantity of magnesium. The success of the experiment was quite surprising, and fully demonstrated the correctness of the hypothesis, since, by the addition of $\frac{1}{2}$ per cent. of magnesium, Fleitmann succeeded in obtaining both nickel and cobalt in a malleable condition. The addition of this small percentage of magnesium appeared to change the structure of these metals entirely and to render them readily weldable when hot. Nickel is even malleable in the cold, while cobalt in the cold becomes extremely hard; and the suggestion is made that the latter metal may be applicable for cutting instruments.

MR. E. CHERNOFF has succeeded in the curious experiment of magnetizing cast iron by surrounding the mould in which the bar was cast by an electro-magnetic reel, along which a current was allowed to flow during the casting process, so that the liquid metal became magnetic and cooled under the influence of the magnetic current. The result was a magnetized bar of white cast iron, but, having a symmetrical cavity extending about two thirds of its length, the metal being extremely thin just opposite the center of the reel. While pouring in the metal, and until it had set, the experimenter had noticed a singular agitation of the metal, and the hollowness of the bar after setting is explained on the theory of the repulsion of the molten metal toward the poles, the agitation above noticed being a visible evidence of this action. It is suggested that powerful and permanent magnets of cast iron may be obtained in this way by casting under pressure, and that, by some modification of the experiment, the plan of magnetizing the fluid metal might be usefully employed in casting hollow cylinders without cores.

THE preparations for raising the huge iron-clad Grosser Kurfürst are now rapidly progressing. The shield for covering the collision hole in her side is ready for use. It is 10 ft. long by 7 ft. broad, oval-shaped and convex. It consists of steel plates $\frac{1}{4}$ inch thick, strongly riveted and strapped together. Two rows of angle-iron, riveted all round the edge of the shield, form a channel for water packing of 6 in. diameter India-rubber hose, which, when filled, will be squeezed with the shield to the ship's side by several very powerful steel screws, bolts and nuts, with very heavy cross-bars inside of hole. A large scaffold for the divers to work on, made of wooden sparring and iron frames, has been constructed. While the extremely difficult and arduous undertaking of attaching the shield to the vessel is going on, various other preparations are being vigorously advanced. The engine for filling the vessel with air is a complicated but very powerful machine. It will be controlled by a Westinghouse governor, in case of sudden mishap during operations to any of its parts. Leading engineers in London have invented a pair of shears for cutting wire rigging under water with the greatest dispatch. When the vessel comes to the surface, by the aid of powerful pontoons, a dozen divers will be engaged in cutting her free from the confused entanglement of wire rope attached to her broken masts and spars, at present buried in the ground under her.

CONEY ISLAND PIER.—The Ocean Navigation and Pier Company, of which Mr. Jacob Lorillard is president, have erected off West Brighton, Coney Island, a large iron pier. The contractors were the Delaware Bridge Company, and the work of construction was under the supervision of Messrs. Maclay and Davies, civil engineers. The pier completed is 1,000 ft. in length, extending outward from high-water mark. Its width is 50 ft., with enlargements of 100 ft. in width at the shore end, the center and the pierhead. It is double-decked, with iron substructure, the whole supported by wrought iron tubular piles, 9 in. in diameter, made of $\frac{1}{2}$ in metal. These piles are arranged in rows, at distances of 20 ft. longitudinally and 16 ft. 8 in. laterally. Each pile has, at its base, a circular cast iron disc $2\frac{1}{2}$ ft. in diameter, which, when sunk into the sand, acts as a supporting base, and at the depth of 15 ft. or 20 ft. insures a perfect foundation. The piles were driven by the "jet water" system. Iron capitals were bolted to the tops of the piles, and they support 15-in. wrought iron beams, bolted together upon which the superstructure rests. The structure is made more secure by being braced throughout with diagonal rods $1\frac{1}{2}$ in. in diameter, and heavy horizontal struts bolted to the beams transversely. The entire structure is supported by 260 iron pillars. The flooring of the lower deck is well finished and enclosed in a handsome iron railing. The landing stage is at the lower deck of the pier head, and guarded by massive oak fender pieces.